Problem Set 3

Physics 445

Due May 17

Some abbreviations: P&S - Peskin & Schroeder

1. Consider the QED Lagrangian with the gauge choice \( \partial_\mu A^\mu = 0 \) and retain the associated ghost terms. Verify that the Lagrangian is invariant under the BRST transformation,

\[
\delta A_\mu = \epsilon \partial_\mu c, \quad \delta \psi = 0, \quad \delta c = 0, \quad \delta \bar{c} = \epsilon \partial_\mu A^\mu.
\]

Construct the associated Noether current and conserved charge. Canonically quantize the theory and express the conserved charge, \( Q \), in terms of creation and annihilation operators for the gauge and ghost fields. Verify the (anti-)commutation relations between \( Q \) and the annihilation/creation operators given in lecture.

2. Do P&S 20.1 for an example of Higgsing.

3. Consider a real scalar field of mass \( m \) coupled to photons,

\[
L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + A\phi \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} F_{\lambda\rho}.
\]

Use this Lagrangian to compute the tree-level decay rate of the scalar field the 2 photons.