1. In typical quantum field theories, we expect generic couplings to be renormalized. There are special cases where a symmetry can protect a coupling from renormalization. The most important of these symmetries is supersymmetry. This symmetry is fermionic, much like the BRST symmetry we studied earlier.

To learn about supersymmetry, consider the following Lagrangian:

\[ L = \phi^* \partial_\mu \partial^\mu \phi - |\lambda|^2 |\phi|^4 - \{ \lambda \psi \bar{\psi} \phi + \text{c.c.} \} - i \psi \sigma^\mu (\partial_\mu \bar{\psi}). \]  

(i) Correcting any misplaced signs or factors of 2, show that the following transformations together with their complex conjugates define a symmetry of the Lagrangian:

\[ \delta_\zeta \phi = \sqrt{2} \zeta \bar{\psi}, \quad \delta_\zeta \psi = i \sqrt{2} \sigma^\mu \zeta \partial_\mu \phi - \sqrt{2} \zeta \lambda^* (\phi^*)^2. \]  

Here \( \zeta_\alpha \) is a Grassmann (fermionic) parameter as in the BRST case, except \( \zeta \) carries a spinor index.

(ii) Let’s carry out a similar renormalization procedure to the one we used in lecture to compute the Yang-Mills \( \beta \)-function. Expand the action around a classical background to quadratic order in quantum fluctuations of the fields. Write down an expression for the 1-loop correction to the \( |\phi|^4 \) coupling. What happens in this case?

2. Consider a real scalar field of mass \( m \) coupled to photons,

\[ L = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + A \phi \epsilon^{\mu \nu \lambda \rho} F_{\mu \nu} F_{\lambda \rho}. \]  

Use this Lagrangian to compute the tree-level decay rate of the scalar field the 2 photons.


4. Oh no! It’s 2-loop time, do P&S 17.1.