Problem Set 2
Physics 445
Due May 12

Some abbreviations: P&S - Peskin & Schroeder


2. Consider the QED Lagrangian with the gauge choice $\partial_\mu A^\mu = 0$ and retain the associated ghost terms. Verify that the Lagrangian is invariant under the BRST transformation,

$$
\delta A_\mu = \epsilon \partial_\mu c, \quad \delta \psi = 0, \quad \delta c = 0, \quad \delta \bar{c} = \epsilon \partial_\mu A^\mu.
$$

Construct the associated Noether current and conserved charge. Canonically quantize the theory and express the conserved charge, $Q$, in terms of creation and annihilation operators for the gauge and ghost fields. Verify the (anti-)commutation relations between $Q$ and the annihilation/creation operators given in lecture.

3. The usefulness of differential forms: let’s consider a $p$-form

$$
\omega = \frac{1}{p!} \omega_{i_1, \ldots, i_p} dx^{i_1} \cdots dx^{i_p} \equiv \omega_{|i_1, \ldots, i_p|} dx^{i_1} \cdots dx^{i_p}
$$

where vertical bars around a set of indices indicate they are summed only over $i_1 < i_2 \cdots < i_p$. Define the dual form, $\ast \omega$, which is an $n-p$ form in $n$ dimensions as the form with components

$$
(\ast \omega)_{k_1, \ldots, k_{n-p}} = \omega_{|i_1, \ldots, i_p|} \epsilon_{i_1, \ldots, i_p, k_1, \ldots, k_{n-p}}.
$$

Here $\epsilon_{i_1, \ldots, i_n}$ is the totally anti-symmetric Levi-Civita tensor. In QED, define a 1-form gauge potential $A = A_\mu dx^\mu$ with a 2-form field strength $F = \frac{1}{2} F_{\mu\nu} dx^\mu dx^\nu$.

(i) Express $F$ in terms of $A$ using the exterior derivative $d$.

(ii) Write Maxwell’s equations in terms of $d, F,$ and $\ast F$.

(iii) Repeat this exercise for non-abelian gauge theory. Define a Lie-algebra valued 1-form $A^{a}_\mu dx^\mu$ and write $F$ in terms of $d$ and $A$. Express the equations of motion in terms of forms.

4. To get a feel for $\beta$-functions, do P&S 16.2.