The Quantum Physics of Neutron Stars

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1 Introduction

Depending on their mass, stars will spend their lifetimes "burning" different elements in their core, using nuclear fusion reactions to maintain hydrostatic equilibrium and avoid collapsing under gravity. Toward the ends of their lives, stars will run out of these fuels and can no longer maintain this balance. While some of the mass is ejected in the form of a planetary nebula, the core will typically collapse into a white dwarf, supported by electron degeneracy pressure rather than fusion reactions. White dwarfs can collapse further if enough mass is added however, and can potentially create the amazing object known as a neutron star. Neutron stars are city-sized objects with densities of an atomic nucleus, comprised almost exclusively of neutrons. This incredible density, of the order $10^{20}$ grams per cubic meter, gives neutron stars an escape velocity of $1/3$ c, corresponding to a gravitational acceleration $10^{11}$ times stronger than that of Earth. This extreme gravity crushes the neutron star into a near perfect sphere; though they have a radius of around 10 km, their surface varies in altitude by 5 mm or less. While neutron stars are a host to many amazing astrophysical phenomena, due to the fact that they are essentially a gargantuan nucleus we can study them through quantum mechanics, and, in turn, learn new quantum mechanical phenomena through the study of them. In this paper we will first analyze neutron stars through quantum mechanics at different stages in their life: first by exploring the degeneracy pressure that supports the neutron star and its progenitor, secondly by studying the quantum physics of the beta decay that occurs during collapse, and finally by discussing the superfluids that occur in a neutron star, a fundamentally quantum phenomenon. Then we will do the reverse, briefly discussing why neutron stars are useful to physicists not specializing in astrophysicists, examining neutron stars’ use in discovering evidence of vacuum birefringence.

2 Pre-Collapse: White Dwarfs and Degeneracy Pressure

Degeneracy pressure is a consequence of the Pauli exclusion Principle, which Wolfgang Pauli asserted in 1925 in order to better explain observed atomic structure. He argued that no two electrons could share the same quantum state, hence explaining why only two (one for each spin) electrons could occupy each subshell. Since, physicists have realized that this principle in fact applies to all fermions; fermions being particles with half-integer spin such as the electron or neutron. One can consider this a consequence of wave function symmetry/anti symmetry. For instance, say we have two particles, one in state one and the other in state two. Our total state can be written as:

$$|\psi_{\text{total}}\rangle = |\psi_1\rangle|\psi_2\rangle$$

(2.1)
However, we must remember that in quantum mechanics, particles of the same type are identical. We, nor anyone, can know which electron is in which state. Hence, our wavefunction must reflect this, and not change if we switch the states of particles 1 and 2. We can construct two versions of such a wavefunction, one which is symmetric and one which is antisymmetric:

\[ |\psi_{\text{total}}\rangle = |\psi_1\rangle|\psi_2\rangle \pm |\psi_2\rangle|\psi_1\rangle \]  

(2.2)

Now, if we switch the two particles, the overall probability distribution, \[ \langle \psi_{\text{total}}|\psi_{\text{total}}\rangle \] will be the same, as desired. However, notice what happens if the \( \pm \) sign is a \(-\), and \[ |\psi_1\rangle = |\psi_2\rangle \].

\[ |\psi_{\text{total}}\rangle = |\psi_1\rangle|\psi_1\rangle - |\psi_1\rangle|\psi_1\rangle = 0 \]  

(2.3)

So if two particles share a state, and use the antisymmetric wavefunction, the wavefunction is nonexistent. Fermions use this antisymmetric wavefunction, and thus this explains why two fermions cannot share a quantum state.

It is thus clear that two fermions do not want to share a quantum state. This resistance to occupying the same state manifests as a pressure, and this pressure can be explicitly calculated using a combination of quantum and statistical mechanics. First, let’s consider what would happen if we tried to force many fermions into the same quantum state. We could do this by applying an extremely large force (such as the gravity in neutron stars) or taking the temperature of a gas of fermions to absolute zero. We will analyze the latter case to get the degeneracy pressure, as the pressure is the same whether it is the result of high density or low temperature, and the low temperature case is more intuitive to calculate, following the logic of a derivation given in [2].

Consider a 3 dimensional state space, where each axis is the \( k_x, k_y, \) and \( k_z \) wave numbers. As the temperature decreases, the average energy of a particle decreases, meaning it moves towards the origin in this space, as each particle’s energy is given by

\[ E = \frac{\hbar k^2}{2m}. \]  

(2.4)

However, as we saw, two fermions cannot occupy the same quantum state, and as such, no two particles and occupy the same spot in this momentum state. Thus, particles build up around the origin, as temperature decreases. When \( T = 0 \), particles will be in the lowest energy state allowed by quantum mechanics, creating a spherical region called a Fermi Surface surrounding the origin. Because momentum is proportional to wave number, \( p = \hbar k \), this also forms a sphere in momentum space. This will be useful in a moment.

We want to calculate the pressure using the formula:

\[ P = \frac{1}{3} \int_{0}^{\infty} v p n(p) dp \]  

(2.5)

Where \( v \) is the velocity and \( n(p) \) is the number density of particles as a function of \( p \). Assuming non-relativisitic particles (this is not always valid, but we will address this later on) \( v = p/m \). Since we are integrating over a sphere in momentum space, \( n(p)dp = \frac{2}{\hbar^2} n(p) 4\pi p^2 dp \), where the leading constant comes from the quantization of a quantum state space. This holds true until we reach the surface of the sphere, as no particles are outside of this radius. Call this maximum \( p_f \), or the Fermi momentum, related to the Fermi Energy, the maximum energy of
fermions at absolute zero. Using the above, we can integrate the expression for \( n(p) \) from 0 to \( p_f \) to get an expression for the number density of particles, \( n \), in terms of \( p_f \)

\[
n = \int_0^{p_f} n(p) dp = \frac{8\pi p_f^3}{3h^3}
\]

(2.6)

Now we are readt to calculate pressure. Plugging our value for \( n(p) \) into our pressure equation and integrating, we arrive at:

\[
P = \frac{8\pi}{15m\hbar^2}p_f^5 = \frac{\hbar^23^{2/3}}{20m\pi^{2/3}}n^{5/3}
\]

(2.7)

Where to arrive at the second result, we have plugged in our relationship between \( n \) and \( p_f \). Thus, we have arrived at an incredibly rudimentary equation of state for neutron stars or white dwarfs, \( P \propto \rho^{5/2} \). We could repeat this process for ultra-relativistic particles, by replacing \( v \) with \( c \). Doing so would have us arrive at \( P \propto \rho^{4/3} \). In white dwarfs, the particles tend to move at around 0.25 \( c \), and are thus not quite ultrarelatvistic, but the relativistic effects also cannot be ignored, and hence the real equation of state for a white dwarf lies somewhere between the two examples we have given. As we will see shortly, a neutron star is also held up by degeneracy pressure, however its degeneracy pressure is derived from neutrons rather than electrons. Similarly, relativistic effects cannot be ignored for a neutron star. However, it is not due to the velocity of the neutrons but rather the incredibly extreme gravity making General Relativity necessary, and thus relativistic effects must be taken into account. Thus, the above equations form a rudimentary model for the equation of state of a neutron star, however a more advanced equation of state is still a subject of cutting-edge research.

### 3 The Collapse: Beta Decay and Fermi’s Golden Rule

So far, we currently have a white dwarf supported by electron degeneracy pressure. Thanks to the efforts of Subrahmanyan Chandrasekhar we know that this arrangement is only gravitationally stable up to about 1.4 solar masses. We are interested in the case in which a white dwarf is suddenly forced high above this mass, such as through a merger with another astronomical object. At this point, the degeneracy pressure cannot support the huge amount of gravity, and the white dwarf begins to collapse. As incredible amounts of mass are forced into smaller spaces, density skyrockets to the point where electron capture beta decay occurs, undergoing the reaction:

\[
e^- + p^+ = n + \nu_e.
\]

(3.1)

How do we model this quantum mechanically? We can use an incredibly useful result of perturbation theory called Fermi’s golden rule. Though named after Enrico Fermi, most of the derivation of this formula was done by Paul Dirac twenty years prior. The golden rule is a formula that describes the transition probability from one energy eigenstate of some unperturbed hamiltonian to a group of energy eigenstates in continuum as the result of a perturbation. Thus, this is perfectly equipped to be used to analyze transition probabilities for beta decay, which was one of its first applications. We can understand the rule through time dependent perturbation theory, following derivations in [10] and [12]. Let us first pretend that the final states, \( \{f\} \) are discrete, and we will impose their continuity later on. We know from basic quantum mechanics that the wave function in the perturbed system can be written as a sum of states. However now that we are dealing with time dependent perturbations, the wave
functions will have time dependent constants $c_n(t)$ as follows:

$$|\psi(t)\rangle = \sum_n c_n(t)e^{iE_n t/\hbar}|\psi_n\rangle$$  \hspace{1cm} (3.2)$$

Considering the time dependent Schrödinger equation, and expanding the Hamiltonian to first order (i.e. $H'$) we can rewrite the equation we need to solve by subtracting all the elements of the equation to one side:

$$(H^0 + H' - i\hbar \frac{d}{dt}) \sum_n c_n(t)e^{iE_n t/\hbar}|\psi_n\rangle = 0$$ \hspace{1cm} (3.3)$$

Now we have a system of differential equations that we wish to use to solve for $c_n$. By applying the time derivative and exploiting the fact that $|\psi_n\rangle$ is already an eigenket of $H^0$ we can write out the differential equation for one of the $c_n$, call it $c_k$:

$$i\hbar \frac{\partial c_k(t)}{\partial t} = \sum_n \langle \psi_k|H'|\psi_n\rangle c_n(t) e^{i(E_k-E_n)t/\hbar}$$ \hspace{1cm} (3.4)$$

Now we begin applying perturbation theory. If $H'$ were zero, then $c_n(t)$ would be $\delta_{n_i}$, representing the fact that the wavefunction would not change from its initial state $|\psi_i\rangle$ as there is nothing to perturb it out of this state. We’ll therefore use $\delta_{n_i}$ as our zeroth order approximation for $c_n(t)$. To get the first order component, we’ll plug $\delta_{n_i}$ into our above equation. The $\delta_{n_i}$ eliminates all but one term of the sum, leaving us with:

$$i\hbar \frac{\partial c_k(t)}{\partial t} = \langle \psi_k|H'|\psi_s\rangle e^{i(E_k-E_s)t/\hbar}$$ \hspace{1cm} (3.5)$$

We can integrate both sides analytically to arrive at an answer:

$$i\hbar c_k(t) = 2\langle \psi_k|H'|\psi_s\rangle e^{i\omega t/2} \frac{\sin(\omega t/2)}{\omega}$$ \hspace{1cm} (3.6)$$

Where $\omega$ has been defined as $(E_k - E_s)/\hbar$ for brevity. The transition rate is the the change in probability of being in a state $|\psi_k\rangle$ with time. The probability of being in any given state is the modulus squared of its leading constant, from basic quantum mechanics. Therefore we have found the transition rate:

$$\lambda_{if} = \frac{d}{dt}|c_k(t)|^2 = \frac{2\langle \psi_k|H'|\psi_s\rangle \sin(\omega t)}{\hbar^2} \frac{\omega}{\omega}$$ \hspace{1cm} (3.7)$$

We are very close now! However, we made the erroneous assumption at the beginning of this analysis that the resulting states will be discrete, but they will in fact form a continuum in the electron capture case. Thankfully, we can get from the discrete result to the continuous result with little effort. We simply have to integrate over their energies times their density of states, $\rho(\omega)$, as follows.

$$\lambda_{if} = \frac{2}{\hbar} \int_{-\infty}^{\infty} \rho(\omega) \langle \psi_k|H'|\psi_s\rangle \sin(\omega t) d\omega$$ \hspace{1cm} (3.8)$$

We can use some useful mathematical tricks to simplify this integral. First of all, for small $\omega$, the integrand is strongly dominated by the $\sin(\omega t)$ function, and hence $\rho(\omega)$ can be pulled out as a constant by comparison. We also assume the matrix element containing the perturbation
can be approximated as constant in time. Then, the only function left in the integrand is \[ \frac{\sin(\alpha)}{\alpha} \], which evaluates to \( \pi \). Therefore we have arrived at our final result, Fermi’s Golden Rule:

\[
\lambda_{if} = \frac{2\pi}{\hbar} |\langle \psi_f | H' | \psi_i \rangle|^2 \rho(E)
\]  
(3.9)

Where \( \lambda_{if} \) is the transition probability, \( |i\rangle \) is the initial state, \( |f\rangle \) are the final states, \( H' \) is the perturbation, and \( \rho(E) \) is the density of states between \( E \) and \( E + dE \) at the energy of the final states.

We can now apply this rule to the specific case of electron capture beta decay in a white dwarf that is collapsing into a neutron star. In this case, the perturbation only occurs on contact, so our perturbation Hamiltonian can be written as:

\[
H' = G_F \delta(r - r_e) \delta(r - r_p)
\]  
(3.10)

where \( G_F \) is a constant that must be determined experimentally, and \( r_p \) and \( r_e \) are the locations of a proton and electron we are considering. If we include the wavefunction for the electron, proton, neutron, and neutrino, this gives a matrix element of:

\[
\langle \psi_f | H' | \psi_i \rangle = G_F \int \psi_{\text{neutrino}}(r) \psi_{\text{neutron}}(r) \psi_{\text{electron}}(r) \psi_{\text{proton}}(r) dr
\]  
(3.11)

This can be rewritten in the form of Fermi’s Ansatz:

\[
\langle \psi_f | H' | \psi_i \rangle = \frac{G_F \mathcal{M}}{V}
\]  
(3.12)

Where \( \mathcal{M} \) is the solution to the integral, which tends to be of order unity, according to Fermi. \( G_F \) is officially dubbed the Fermi constant, and \( V \) is a normalization volume. This can then be plugged into Fermi’s golden rule, but we need an expression for density of states as well. The density of states can be found from the product of the neutron and neutrino phase space volumes, which each carry a factor of \( V \) which nicely cancel the \( V^2 \) introduced from the square of the matrix element. Using the same method as in section one, we can integrate over spherical shells in momentum space, replacing \( dp^3 \) with \( 4\pi p^2 dp \). We can convert the neutrino phase space volume to be in terms of energy, giving a final density of states of:

\[
\rho(E_0) = \frac{1}{4\pi^2 \hbar^2 c^3} p^2 (E_0 - E)^2 dp
\]  
(3.13)

This, along with the matrix element, can be plugged into Fermi’s Golden Rule to arrive at a transition probability for Beta Decay:

\[
\lambda_{if} = \frac{G_F^2 |\mathcal{M}|^2}{2\pi^3 \hbar^2 c^3} p^2 (E_0 - E)^2 dp
\]  
(3.14)

If we like, this can be integrated to yield a transition rate, where I am making the assumption that the electrons are relativistic, considering they have velocities of 0.25 \( c \) in a collapsing white dwarf:

\[
W = \frac{G_F^2 |\mathcal{M}|^2 Q^5}{60\pi^3 \hbar^2 c^6}
\]  
(3.15)

Where \( Q = E_0 - (m_c c^2) \). This result is valid for 15 orders of magnitude of Beta Decay, including our case, and is valid for cases of nuclei and elementary particles alike! Thus we have arrived at a rudimentary mathematical model, based on quantum mechanics and statistical mechanics, for rates and probabilities of beta decay inside a forming neutron star. This model is enough to show us that under the conditions of a neutron star collapse, roughly 90% of the resulting neutron star will be neutrons. This result is important as the fact that there are some protons left over (~5% of the neutron star by number) will explain why the superfluid in a neutron star is superconducting, as we will discuss in the next section.
Quantum Fluids in the Neutron Star Interior

As we saw in the introduction and section one, the incredible gravity as a result of a neutron star’s extreme density makes for some profoundly interesting physics. Another example of this is the superfluidity and superconductivity of the particle sea within the neutron star. Superfluidity means that the fluid flows without viscosity or friction, while superconductivity means that charges can move through the material without resistance. In order to understand these fascinating states of matter within the neutron star, we must delve into quantum mechanics and condensed matter physics.

Superfluids are more commonly constructed of bosons rather than fermions. However, a neutron star, being made of neutrons, is comprised of solely fermions. However, it will be useful to first study bosonic superfluids, and then look at the fermionic case.

Superfluidity was first discovered in liquid Helium II in 1938, below a temperature of 2.17 K it was discovered to flow with practically zero viscosity. The key to this is the fact that Helium forms a Bose-Einstein Condensate at this low of a temperature. Recall that in section one when we discussed fermions, they form a Fermi surface at low temperatures or high pressures due to the Pauli Exclusion Principle. For bosons, this principle is not valid, (they use the $+$ sign in equation 2.2) and they can all sink to the lowest possible energy quantum state. As a result, in a BEC quantum mechanical effects become important macroscopically. A BEC can carry charge and flow without losing energy, but any excitations out of the lowest energy quantum state can potentially introduce dissipation that ruins superfluidity. Lev Landau derived a useful argument for when this occurs. If $\varepsilon_p$ is the energy of an excitation with a momentum of $\vec{p}$, then the fluid will lose energy due to dissipation if

$$\varepsilon_p + \vec{p} \cdot \vec{v}_s < 0. \quad (4.1)$$

Here, $\vec{v}_s$ is the velocity of the tube that the fluid is moving through, if we are in the rest frame of the fluid. Therefore, the fluid does not lose energy to dissipation (and is hence a superfluid) when the fluid is moving slower than a critical velocity:

$$\vec{v}_s = \min(\varepsilon_p/\rho) \quad (4.2)$$

If this critical velocity is zero, then the fluid cannot be superfluid. If it is nonzero, and the fluid is moving slower than this critical velocity, the Bose-Einstein condensate is not excited out of its ground state and the fluid flows without viscosity!

Now we have a basic understanding of superfluidity in bosonic material, but what about fermionic material, like that in neutron stars? A fundamental concept to understand for superfluidity is that of Cooper Pairing. Cooper Pairing is a phenomenon by which electrons, which would normally repel each other due to their shared negative charge, experience a slight attraction due to the exchange of a quasiparticle called a phonon. This concept is fundamentally quantum, but it has a useful classical analog to help get a basic understanding. If we consider a free electron in a metal, it will repel other electrons but attract the positive ions that make up the lattice. As the electron pulls positive charges towards itself, the density of positive charge increases. At large distances, another electron can be attracted to this region of increased positive charge, overcoming the electron’s mutual repulsion. Now the two electrons can be considered paired. There are two important results of this. Firstly, as a fermion, each electron has spin 1/2, so the total spin of the electron pair must be 0 or 1, making it bosonic. It is this phenomenon that allows for superfluidity, as the electrons are now behaving as though they
were bosons! However, in order to fully understand the mechanics of Cooper Pairing we must discuss BCS Theory and the gap function.

BCS theory is short for Bardeen–Cooper–Schrieffer theory and was the first microscopic theory of superfluidity/conductivity. This theory won the Nobel Prize for Physics in 1972. The following derivation uses logic from derivations given in [7] and [17]. The BCS theory describes the combined state for two spin-1/2 particles in a spin singlet state with total angular momentum 0 interacting under a two-body potential as follows:

$$|\psi\rangle = \prod_k (u_k + v_k c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger)|0\rangle$$ (4.3)

Here, $u_k$ and $v_k$ are just constants that are real and positive, with $u_k^2 + v_k^2 = 1$ while the $c_{k\uparrow}^\dagger$ creates a particle with wavenumber $k$ and spin $\uparrow$ or $\downarrow$ while $c_{k\downarrow}$ destroys one. This wavefunction is not an eigenstate of the total number of particles, $\hat{N}$, so we will work in the grand canonical ensemble from statistical mechanics. When using the BCS theory's approximations, we work with the expectation value of the Hamiltonian. Since we are working in the grand canonical ensemble this means we must also find the expectation value of $\hat{N}$, as follows:

$$\langle N \rangle = \langle \psi_{BCS} | \sum_k (c_{k\uparrow}^\dagger c_{k\uparrow} c_{k\downarrow}^\dagger c_{k\downarrow}) | \psi_{BCS} \rangle = 2 \sum_k u_k^2$$ (4.4)

Where $\psi_{BCS}$ is the wave function we found above, and the sum in the center is $\hat{N}$ in terms of the creation and annihilation operators. Having this we can now solve for the expectation value of the Hamiltonian as required by BCS theory. Terms in this that do not correspond to pairing interactions have the form $v_k^2 u_k^2$ and are neglected. This gives us:

$$\langle H - \mu \hat{N} \rangle = 2 \sum_k \xi(k) u_k^2 + \sum_{kk'} \langle k | V | k' \rangle u_k v_k u_{k'} v_{k'} \langle k' | V | k \rangle$$ (4.5)

where we have defined $\xi(k)$ as $\hbar^2 k^2 / 2m - \mu$, i.e. the kinetic energy of a particle with wavevector $k$ minus the chemical potential. The matrix element $\langle k | V | k' \rangle$ is the matrix element for the interaction hamiltonian between a state of two particles with opposite spin with wavevectors $\pm k$ and the same but with wavevectors $\pm k'$. We can take advantage of the fact that $u_k^2 + v_k^2 = 1$ and reduce this equation, giving us a very important result, the gap equation:

$$\Delta(k) = -\sum_{k'} \langle k | V | k' \rangle \frac{\Delta(k')}{2E(k')}$$ (4.6)

where the "gap" is given by:

$$\Delta(k) = -\sum_{k'} \langle k | V | k' \rangle u_{k'} v_{k'}$$ (4.7)

$E$ is called the "quasiparticle excitation energy" and is given by $E = \sqrt{\xi(k)^2 + \Delta(k)^2}$. This result, along with the expression for average particle number, which we can now write as:

$$\langle N \rangle = \sum_k \left[ 1 - \frac{\xi(k)}{E(k)} \right]$$ (4.8)

can be used to solve for the chemical potential $\mu$. These equations are coupled in the case of a neutron star (they become uncoupled in the low density limit) and thus generally have to be
solved numerically. Nonetheless, these equations can be used to describe the type of fluids in the neutron star. Numerical calculations for this gap as a function of the Fermi wavenumber (i.e. the wavenumber corresponding to the Fermi Energy. Note that $k_F^1$ is proportional to interparticle separation, and hence proportional to the depth into the neutron star) are shown below:

What is this gap? The actual gap that the gap equation refers to is the energy gap in the quasiparticle spectrum of a Cooper pair. In more understandable terms, it is this gap in the spectrum that allows for superconductivity/fluidity. Hence, the peaks in this figure represent where the neutron star is most superfluid.

At lower densities, i.e. towards the surface of the neutron star, the neutrons form what is called a $^1S_0$ superfluid. This corresponds to the peak on the left half of the figure. The $^1S_0$ symbol is the same as the symbology you might remember from addition of angular momentum in basic quantum mechanics. The top left number is $2s+1$, where s is the total spin of the two particles, the letter is the orbital angular momentum, and the bottom right number is the total angular momentum, J. Here, the symbol refers to the combined angular momentum state of the Cooper paired particles.

Deeper in the stellar core, a different kind of superfluid forms. Instead of pairs forming in the state $^1S_0$, at the extremely high density at the center of a neutron star, Cooper pairs forming in the $^3P_2$ channel is more attractive. There is a small additional coupling in the $^3F_2$ channel due to the tensor force, a spin dependent force between nucleons.

In order to fully understand the superfluid at the center of a neutron star, we must also consider the proton contribution. A neutron star after its collapse is in beta equilibrium, meaning there is an equal rate of neutrons undergoing beta decay to produce an electron and proton, and electron capture, where the reverse happens. This results in a material that is roughly 95% neutrons, and 5% electrons and protons. Due to the fact that there are much fewer protons than neutrons, their Fermi Energy is much lower, which in turn causes protons to achieve Cooper pairing, like the neutrons. While it is commonly accepted that protons reach only $^1S_0$ pairing in a neutron star, it is currently a matter of debate as to whether or not they achieve $^3P_2$ pairing deep in the stellar core. Regardless, this Cooper pairing is important as it gives the superfluid in the center of neutron stars special superconductive properties.
Why is the superfluidity in neutron stars important? Though a quantum phenomenon, superfluidity has effects drastic enough to be observed at an astrophysical scale. Neutron stars have strong magnetic fields, enough to slow down the rotation of the outer crust of the star over time. However, as we’ve demonstrated, the inner regions of a neutron star are superfluid and thus won’t slow in this way. Thus, over time, the crust slows while the interior doesn’t, leading to a large velocity disparity throughout the star. Eventually, this difference becomes great enough that the interior can transfer some of its rotation to the outer crust, causing it to suddenly speed up. This phenomenon is called a neutron star glitch, and can be measured by the change in the rate of the star’s radio pulses as a result in the sudden change of the rotation speed of the crust of the star. These glitches can then be used to determine the mass of the neutron star, allowing us to learn more about these fascinating objects.

5 Applications: Vacuum Birefringence

More practical physicists might be wondering about the importance of studying these objects. After all, the nearest neutron star to us is around 200 light years away, how does studying something so far away advance more pragmatic and useful physics? As it turns out, neutron stars make an excellent laboratory for studying phenomenon that would be difficult to study on Earth. In fact, in 2016 astronomers used the study of neutron stars to prove the existence of Vacuum Birefringence, a phenomenon predicted by quantum electrodynamics 80 years prior but until then had been unobserved. What is Vacuum Birefringence, and why do we see it near neutron stars?

First, we need to take a quick dive into the quantum theory of optics. Quantum mechanics tells us that light is both a beam of protons and a wave constructed from oscillating electric and magnetic fields. Light has different types of polarization depending on how the electric
field vector moves as it oscillates. Light polarized at an arbitrary angle is actually a quantum superposition of a photon polarized horizontally and a photon polarized vertically. If we represent the horizontally polarized photon as \( |h\rangle \) and the vertical as \( |v\rangle \), we can consider these as an orthonormal basis that spans polarization states. Thus light polarized at some angle \( \theta \) can be written as:

\[
|\theta\rangle = |v\rangle \langle v| |\theta\rangle + |h\rangle \langle h| |\theta\rangle
\]

Using a little trigonometry, these projections can be rewritten to:

\[
|\theta\rangle = |v\rangle \cos(\theta) + |h\rangle \sin(\theta)
\]

Light can also be unpolarized, meaning it is in a mixed state of orthogonal polarization bases, and hence cannot be represented by a single wave function. (Due to the fact that the polarization of light can be represented this way shown above means that photons are analogous to qubits!)

Though our human eyes cannot tell the difference between different polarizations of light, but it can affect certain aspects of optical physics. For example, Birefringence is a property of some transparent crystals where the speed of light passing through it depends on its polarization. In normal transparent materials, the transparency is usually due to free electrons in the material. The electric field of the light wave vibrates the electrons in time with the wave. Many electrons moving together in synchronicity in this fashion allows them to capture and then re-emit the light energy, allowing it to pass through the material. Birefringence, then, results when the structure of a crystal gives electrons more freedom of movement in one direction than another. This resistance to movement in particular directions results in the light speed being dependent on the light’s polarization: if the polarization of the light is such that it tries to oscillate electrons in a direction that the crystal structure resists, the light speed will be slower.

If this is Birefringence, then how can this occur in a vacuum? The theory dates back to 1936, when Werner Heisenberg and Hans Euler were studying the effects of strong magnetic and electric fields on light. At around this time, quantum electrodynamics was a new and exciting theory. One of its predictions is that the vacuum is not completely empty: it is actually full of virtual electrons and positrons being spontaneously created and then annihilating each other. Heisenberg and Euler predicted that very strong magnetic fields would affect these particles which could in turn affect light traveling through the vacuum. In particular, at a magnetic field stronger than 4.41 billion Tesla could cause these virtual particles to give the vacuum birefringent properties: different polarizations of light could have very different speeds, differing by a significant fraction of \( c \)!

4.41 billion Tesla is far beyond what is currently capable of being created in a physics laboratory, especially in 1936. It is even greater than the magnetic field surrounding our sun. However, this magnetic field is perfectly achievable by a neutron star, whose magnetic field can range in strength from \( 10^4 \) to \( 10^{11} \) Tesla. This field is generated by currents in the core of the star, but it is not entirely clear why neutron stars have such high magnetic fields. One hypothesis is "flux freezing", where the strength of the magnetic flux is conserved; i.e., magnetic flux is the same before and after the collapse, but the fact that it is spread over a smaller surface area (after the collapse to neutron star) results in a much stronger magnetic field. Another hypothesis is that is related to the \( ^3P_2 \) neutron superfluid we discussed in the previous section; the \( ^3P_2 \) Cooper pairs have a paramagnetic moment that could be boosting the field.

Whatever the reason, the huge magnetic fields due to a neutron star make these regions the perfect (and thus far, only) environment to study vacuum birefringence. However, there is a bit
of a problem. Neutron stars are not very bright. The first discovered neutron stars were actually pulsars, discovered in 1967 by Jocelyn Bell. The problem is that these easier to detect pulsars emit all of this light in the direction of the magnetic field, where QED predicts no vacuum birefringence would occur. Thus, directly observing vacuum birefringence is difficult even around neutron stars. Thankfully, in 2016 astronomers were able to use powerful telescopes to observe the very dim neutron star RX J1856.5-3754. If we assume that the magnetic field of the neutron star is somewhat like a dipole, vacuum birefringence should force light towards a linear polarization. This is exactly what was observed! Light being emitted in the region where vacuum birefringence was expected had a linear polarization of $16.4 \pm 5.3\%$. This unfortunately, is not enough to be complete and utter proof of the phenomenon, as there are a few (quite unlikely) other processes that could also cause these observations. However, it is still a good sign for quantum electrodynamics in general, and hints toward possible future experiments where more obvious signs of vacuum birefringence could be spotted around magnetars. In addition, the Imaging X-ray Polarimetry Explorer which will be launched in 2021 will carry equipment perfectly equipped for observing vacuum birefringence, and can hopefully push us past just indirect evidence of this fascinating phenomenon.

6 Conclusion

Neutron stars are a great example of the vast interconnectedness of physics: through the study of quantum mechanics we can learn a great deal about astrophysical objects, and vice versa. We have hardly scratched the surface of all of the physics contained within neutron stars, and we have already covered a vast subsection of quantum mechanics: Pauli Exclusion Principle and the Degeneracy Pressure, Fermi’s Golden Rule / Time dependent perturbation theory and its applications to beta decay, the BCS Theory, Cooper Pairs and their applications to superfluidity, as well as the quantum nature of light. In turn, neutron stars can teach us about quantum mechanics as well, being amazing laboratories to observe some of the effects of quantum mechanics occurring in extreme environments, such as vacuum birefringence. If we continue to apply quantum mechanics and other far reaching areas of physics to astronomical objects such as neutron stars, there is certainly much more we can learn about how our fascinating universe works.

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