1. Let’s get more comfortable with quantum circuits. In the basis $|00⟩, |01⟩, |10⟩, |11⟩$, often called the computational basis because it involves eigenstates of $Z$, the CNOT gate has the matrix representation:

$$
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{pmatrix}.
$$

What is the analogous matrix representation for the following two circuits in the same basis? Are the matrices unitary?

2. Show that:

3. Consider a single qubit operator $U$ that is unitary but also Hermitian with eigenvalues $\pm 1$. It is both a gate and an observable. Show that the following circuit measures the observable $U$ in the following sense: we want a measurement result that indicates one of the two eigenvalues of $U$ and leaves $|\psi_{out}\rangle$ in a state corresponding to that eigenvector.

4. Lastly, we will use this result in our class discussion. Let $V$ be a Hilbert space with subspace $W$. Suppose $U : W \rightarrow V$ is a linear operator that preserves inner products. Show that there exists a unitary operator $U' : V \rightarrow V$ with the property that $U' |w⟩ = U |w⟩$ for $|w⟩ \in W$ but with $U'$ defined on all of $V$. 
