Holographic Inflation confronts data

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The talk is based on

- P. McFadden, KS, Cosmological 3-point correlators from holography, arXiv:1104.3894

Earlier work with Paul McFadden

- Holography for Cosmology, arXiv:0907.5542
- Observational signatures of holographic models of inflation, arXiv:1010.0244
- Holographic Non-Gaussianity, arXiv:1011.0452

Related work

- M. Dias, Cosmology at the boundary of de Sitter using the dS/QFT correspondence, arXiv:1104.0625
Any gravitational theory should be **holographic**, i.e. it should have a description in terms of a **non-gravitational** theory in one dimension less.

There is no holographic construction that works in general to date; explicit examples depend on the **form of asymptotics**.

The **properties** of the dual theory depend also on the **asymptotics**.

The best-understood examples originate from string theory via decoupling limits of branes:

- D3, M5 etc. branes $\rightarrow$ asymptotically AdS spacetimes $\rightarrow$ local QFT that in the UV becomes **conformal**.
- D2, D4 etc. $\rightarrow$ asymptotically power-law spacetimes $\rightarrow$ local QFT with a **generalized conformal structure**.
Here I will describe a holographic framework for inflationary spacetimes that:

1. approach de Sitter spacetime at late times,

\[ ds^2 \rightarrow ds^2 = -dt^2 + e^{2t} dx^i dx^i, \quad \text{as} \quad t \rightarrow \infty \]

2. approach power-law scaling solutions at late times,

\[ ds^2 \rightarrow ds^2 = -dt^2 + t^{2n} dx^i dx^i, \quad (n > 1) \quad \text{as} \quad t \rightarrow \infty \]

These backgrounds have the property that they are in 1-1 correspondence with backgrounds that describe holographic RG flows, either asymptotically AdS or asymptotically power-law.
This correspondence between inflationary spacetimes and holographic RG flows is a special case of a more general Domain-wall/Cosmology correspondence (DW/C) [KS, Townsend (2006)] and can be understood as analytic continuation.

For holographic RG flows, there is a well-established holographic dictionary.

One may express the analytic continuation associated with DW/C in QFT terms: one analytically continues the momenta $q$ and the rank of the gauge group $N$:

$$q \rightarrow \bar{q} = -iq, \quad N \rightarrow \bar{N} = -iN$$
Holographic framework [McFadden, KS (2009)]

Holographic RG Flow \[\rightsquigarrow\] QFT

\[\uparrow\quad\text{Gauge/Gravity duality}\]

\[\downarrow\quad\text{Domain Wall/Cosmology correspondence}\]

Cosmology \[\leftrightarrow 'pseudo'-QFT\]

\[\uparrow\quad\text{Analytic continuation}\]
To test this framework we analyzed the case where gravity is weakly coupled, so standard treatments are available.

- We worked out the cosmological 2- and 3-point functions of scalar and tensor perturbations around a general single scalar inflationary background.
- We worked out the 2- and 3-point functions of the stress energy tensor of the corresponding domain-wall background, using standard gauge/gravity duality.
- We found that the cosmological correlators can be expressed in terms of the QFT correlation functions at strong coupling upon analytic continuation.

⇒ Standard inflation is holographic.
The 2-point function of $T_{ij}$ in a flat spacetime has the form

$$\langle T_{ij}(\bar{q}) T_{kl}(-\bar{q}) \rangle = A(\bar{q}) \Pi_{ijkl} + B(\bar{q}) \pi_{ij} \pi_{kl},$$

where $\Pi_{ijkl} = \frac{1}{2} (\pi_{ik} \pi_{lj} + \pi_{il} \pi_{kj} - \pi_{ij} \pi_{kl})$, $\pi_{ij} = \delta_{ij} - \bar{q}_i \bar{q}_j / \bar{q}^2$.

We found useful to use a helicity basis.

$$T_{ij} \rightarrow T, T^{(s)}, \quad s = \pm 1$$

where $T$ is the trace of $T_{ij}$ and the transverse traceless part of $T_{ij}$ is traded for $T^{\pm}$.

In cosmology, the physical degrees of freedom are a scalar mode, $\zeta$, and transverse traceless tensor modes $\hat{\gamma}_{ij}$. In a helicity basis:

$$(\delta \phi, h_{ij}) \rightarrow \zeta, \hat{\gamma}^{(s)}, \quad s = \pm 1$$
The cosmological 2-point functions are given by

\[ \langle \zeta(q)\zeta(-q) \rangle = \frac{-1}{8\text{Im}[B(q)]} , \quad \langle \hat{\gamma}^{(s)}(q)\hat{\gamma}^{(s')}(q) \rangle = \frac{-\delta^{ss'}}{\text{Im}[A(q)]} , \]

The 2-point functions determine the power spectra (measured by WMAP and other experiments)

\[ \Delta^2_R(q) = \frac{q^3}{2\pi^2} \left( \frac{-1}{8\text{Im}[B(-iq)]} \right) , \quad \Delta^2_T(q) = \frac{2q^3}{\pi^2} \left( \frac{-1}{\text{Im}[A(-iq)]} \right) , \]

[McFadden, KS (2009)]
Holographic formulae: 3-point functions

1. \[ \langle \zeta(q_1)\zeta(q_2)\zeta(q_3) \rangle \]
   \[ = -\frac{1}{256} \left( \prod_i \text{Im}[B(\bar{q}_i)] \right)^{-1} \times \text{Im} \left[ \langle T(\bar{q}_1)T(\bar{q}_2)T(\bar{q}_3) \rangle + (\text{semi-local terms}) \right], \]

2. \[ \langle \zeta(q_1)\zeta(q_2)\hat{\gamma}^{(s_3)}(q_3) \rangle \]
   \[ = -\frac{1}{32} \left( \text{Im}[B(\bar{q}_1)]\text{Im}[B(\bar{q}_2)]\text{Im}[A(\bar{q}_3)] \right)^{-1} \]
   \[ \times \text{Im} \left[ \langle T(\bar{q}_1)T(\bar{q}_2)T^{(s_3)}(\bar{q}_3) \rangle + (\text{semi-local terms}) \right], \]

[McFadden, KS (2010), (2011)]
Holographic formulae: 3-point functions

- $\langle \zeta(q_1) \hat{\gamma}^{(s_2)}(q_2) \hat{\gamma}^{(s_3)}(q_3) \rangle$

  $= -\frac{1}{4} \left( \text{Im}[B(\bar{q}_1)] \text{Im}[A(\bar{q}_2)] \text{Im}[A(\bar{q}_3)] \right)^{-1}$

  $\times \text{Im} \left[ \langle T(\bar{q}_1) T^{(s_2)}(\bar{q}_2) T^{(s_3)}(\bar{q}_3) \rangle + \text{(semi-local terms)} \right]$, 

- $\langle \hat{\gamma}^{(s_1)}(q_1) \hat{\gamma}^{(s_2)}(q_2) \hat{\gamma}^{(s_3)}(q_3) \rangle$

  $= - \left( \prod_i \text{Im}[A(\bar{q}_i)] \right)^{-1} \times \text{Im} \left[ 2 \langle T^{(s_1)}(\bar{q}_1) T^{(s_2)}(\bar{q}_2) T^{(s_3)}(\bar{q}_3) \rangle + \text{(semi-local terms)} \right]$

[McFadden, KS (2011)]
Remarks

The cosmological 3-point functions are related with non-gaussianities and the Planck satellite (with data expected to be released in 2012) and other missions are expected to constrain them significantly.

The semi-local terms correspond to contributions where two of the three insertion points in the 3-point function are coincident. Such terms contribute to the so-called local type non-gaussianity.

The holographic formulae are consistent with the interpretation of such duality as computing the wavefunction of the universe, as discussed for de Sitter by [Maldacena (2002)].

The non-gaussianity for tensor modes in exactly de Sitter background computed recently in [Maldacena, Primentel (2011)] is in agreement with our general formulae.
While conventional inflationary models are related with strongly coupled QFT, new models arise when we consider the QFT at weak coupling (but still at large $N$).

In these models, the very early universe is in a non-geometric phase. This phase should have a description in string theory in terms of a strongly coupled sigma model. Here we use holography to describe it.

The end of this period is the beginning of hot big bang cosmology.
To make predictions we need to specify the dual QFT. The two classes of asymptotic behaviors correspond to two classes of dual QFT’s.

- asymptotically de Sitter → QFT is deformation of a CFT
- asymptotically power-law → QFT has generalized conformal structure

Here we discuss theories of the second type.
We require that the theory has the following properties:

1. admits a large $N$ limit
2. all fields are massless
3. it has a dimensionful coupling constant
4. all terms in the Lagrangian have the same scaling dimension, which should be different from three.

Properties (2)-(4) imply that the theory admits a generalized conformal structure: the theory would be conformal if the coupling constant is promoted to a background field that transforms under conformal transformations [Jevicki et. al. (1998)].
A class of models exhibiting these properties is given by the following super-renormalizable theory:

\[ S = \frac{1}{g_{\text{YM}}^2} \int d^3x \text{tr} \left[ \frac{1}{2} F_{ij}^I F^{ij} + \frac{1}{2} (D \phi^I)^2 + \frac{1}{2} (D \chi^K)^2 + \bar{\psi}^L D \psi^L \right. \\
\left. + \lambda_{M_1 M_2 M_3 M_4} \Phi^M_{M_1} \Phi^M_{M_2} \Phi^M_{M_3} \Phi^M_{M_4} + \mu_{\alpha \beta}^{L_1 L_2} \Phi^M_{L_1} \psi_{\alpha}^L \psi_{\beta}^L \right]. \]

\( \Phi^M = \{ \phi^I, \chi^K \} \), \( \chi^K \): conformal scalars, \( \phi^I \): minimally coupled scalars, \( \psi^L \): fermions

To extract predictions we need to compute \( n \)-point functions of the stress energy tensor analytically continue the result and insert them in the holographic formulae.

We have computed the leading order result both for 2- and 3-point functions [Bzowski, McFadden, KS] (to appear). Here I will focus on the scalar power spectrum since it is this case where comparison with data is currently possible.
The relevant correlation function is the 2-point function of the trace of the stress energy tensor $T$. The form of this 2-point function is fixed by generalized conformal structure [Kanitscheider, KS, Taylor (2008)]. At large $N$,

$$\langle T(q)T(-q) \rangle = q^3 N^2 f(g_{\text{eff}}^2),$$

where $g_{\text{eff}}^2 = g_{YM}^2 N/q$ is the effective dimensionless 't Hooft coupling and $f(g_{\text{eff}}^2)$ is a general function of $g_{\text{eff}}^2$.

The analytic continuation acts as

$$g_{\text{eff}}^2 \rightarrow g_{\text{eff}}^2, \quad N^2 q^3 \rightarrow -iN^2 q^3$$

and therefore

$$\Delta_R^2(q) = -\frac{q^3}{4\pi^2} \frac{1}{\text{Im}\langle T(q)T(-q) \rangle} = \frac{1}{4\pi^2 N^2} f(g_{\text{eff}}^2)$$

It remains to compute $f(g_{\text{eff}}^2)$ ...
When $g_{\text{eff}}^2$ is small, one finds that the function $f(g_{\text{eff}}^2)$ has the form

$$f(g_{\text{eff}}^2) = f_0(1 - f_1 g_{\text{eff}}^2 \ln g_{\text{eff}}^2 + f_2 g_{\text{eff}}^2 + O[g_{\text{eff}}^4]).$$

→ $f_0$ is determined at 1-loop in perturbation theory. It has been computed in [McFadden, KS (2009)].

→ $f_1$ is determined at 2-loop in perturbation theory. It has not been computed to date.

→ $f_2$ is related with an infrared generated scale $q_{\text{IR}} \sim g_{\text{YM}}^2$ [Jackiw, Templeton (1981)] [Appelquist, Pisarski (1981)]. As long as one probes the theory at scales large compared with the IR scale, this term is negligible.
Redefining variables, \( f_1 g_{YM}^2 N = gq_* \), where \( q_* \) is a reference scale that is taken to be \( q_* = 0.05 \text{ Mpc}^{-1} \) (the WMAP momentum range is \( 10^{-4} \lesssim q \lesssim 10^{-1} \text{ Mpc}^{-1} \)), we obtain the final formula:

\[
\Delta^2_R(q) = \Delta^2_R \left( \frac{1}{1 + \frac{(gq_*)}{q} \ln \left| \frac{q}{gq_*} \right|} \right),
\]

\( \rightarrow \) \( \Delta^2_R = 1/(4\pi^2 N^2 f_0) \). Smallness of the amplitude is related with the large \( N \) limit: matching with observations implies \( N \sim 10^4 \).

\( \rightarrow \) When \( (gq_*)/q \ll 1 \) one may rewrite the spectrum in the power-law form

\[
\Delta^2_R(q) = \Delta^2_R q^{n_s - 1}, \quad n_s(q) - 1 = gq_*/q
\]

Thus the small deviation from scale invariance appears to be related with the coupling constant of the dual QFT being small.
Holographic power spectrum

Blue curve: $g > 0$, Red curve: $g < 0$
This is a rather different spectrum than that of generic slow-roll models. In such models the dependence of $n_s$ on $q$ is rather weak. The "running" $\alpha_s = dn_s/d \ln q$ is higher order in slow-roll than $(n_s - 1)$,

$$\alpha_s/(n_s - 1) \sim \epsilon$$

In contrast, in the holographic model, $\alpha_s \sim (n_s - 1)$. In fact, all $d^k n_s(q)/d \ln q^k$ are of the same order.

Given that the power-law $\Lambda$CDM model, in which $n_s$ is a constant, fits remarkably well the WMAP and other astrophysical data, one may wonder whether already existing data are sufficient to rule out this class of holographic models.

We thus undertook the task to make a dedicated data analysis. [Easther, Flauger, McFadden, KS (2011). Related work appeared in [Dias (2011)]].
The power-law ΛCDM model depends on six parameters. Four describe the composition and expansion of the universe and the other two are the tilt $n_s$ and the amplitude $\Delta^2_R$ of primordial curvature perturbations.

The holographic ΛCDM model depends on the same set of parameters, except that the tilt $n_s$ is replaced by the parameter $g$.

We determined the best-fit values for all parameters for both models and used Bayesian evidence in order to make a model comparison.
Angular power spectrum: $\Lambda$CDM vs holographic model

Red: $\Lambda$CDM, Green: holographic model
The estimated values for the five common parameters of the two models are roughly within one standard deviation of each other.

The data favor negative values of $g$ (red spectrum) with central value $g = -1.27 \times 10^{-3}$.

This indeed leads to a small effective coupling, except potentially for the very low wavelength modes. Since $g_{\text{eff}}^2 = (1/f_1)(gq_*/q)$ one needs to know the value of the 2-loop factor $f_1$ when $(gq_*/q)$ itself is not very small.

A related issue is that the infrared scale $q_{\text{IR}}$ may be inside the WMAP range. In such case the precise value of the parameter $f_2$ is important and the power spectrum is modified:

$$\Delta_R^2(q) = \Delta_R^2 \frac{1}{1 + (gq_*/q) \ln |q/\beta gq_*|}$$

If $g_{\text{eff}}^2$ is not small for all relevant momenta, one must include higher order terms in the computation of the 2-point function.

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The holographic model is compatible with current data, but the overall fit is somewhat better for $\Lambda$CDM:

<table>
<thead>
<tr>
<th></th>
<th>Holographic Model</th>
<th>$\Lambda$CDM</th>
<th>$\Delta \ln L_{\text{best}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>WMAP7</td>
<td>3735.5</td>
<td>3734.3</td>
<td>1.2</td>
</tr>
<tr>
<td>WMAP+BAO+$H_0$</td>
<td>3737.3</td>
<td>3735.7</td>
<td>1.6</td>
</tr>
<tr>
<td>WMAP+CMB</td>
<td>3815.0</td>
<td>3812.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table: Best-fit values for $-\ln L$ for both the holographic model and $\Lambda$CDM, as well as the difference between them. Positive numbers in the last column favor $\Lambda$CDM.

One often uses the value of the likelihood at the best-fit point as the criterion for a model selection. However, this is the probability for obtaining the data given a model with specific parameter values.

For model comparison we would like to know the probability for a model given the data.
The probability for a model given the data is computed by the Bayesian evidence $E$,

$$ E = \int d\alpha M P(\alpha_M) L(\alpha_M) $$

$P(\alpha_M)$ is the probability that a choice of parameters $\alpha_M$ is realized and $L(\alpha_M)$ is the probability for the data given these parameters.

A rough guide for model selection is Jeffreys scale:

<table>
<thead>
<tr>
<th>$\Delta \ln E$</th>
<th>Strength of evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 1$</td>
<td>Inconclusive</td>
</tr>
<tr>
<td>$&gt; 1$</td>
<td>weak evidence</td>
</tr>
<tr>
<td>$&gt; 2.5$</td>
<td>Moderate evidence</td>
</tr>
<tr>
<td>$&gt; 5$</td>
<td>Strong evidence</td>
</tr>
</tbody>
</table>
The computation of the evidence depends on the prior probability $P(\alpha_M)$. The priors ought to reflect the underlying assumptions and knowledge of the problem before the data came along.

For the holographic model, the prior for $g$ is clear: the power spectrum was obtained from a perturbative computation $\rightarrow g$ must not be very large.

$\Lambda$CDM is an empirical model and the choice of priors for $n_s$ is more subjective. We considered two choices: (i) a nearly optimal choice for $\Lambda$CDM, $0.92 < n_s < 1$, and (ii) a symmetric choice around the scale invariant spectrum $0.9 < n_s < 1.1$.

With choice (i) there is a weak evidence for $\Lambda$CDM. With choice (ii) the evidence is inconclusive.

More data is required to decide between the two models.
Solid line: $\alpha = - (n_s - 1)$
Conclusions

- **Standard inflation is holographic:** standard observables such as power spectra and non-Gaussianities can be expressed in terms of (analytic continuation of) correlation functions of a dual QFT.

- There are **new holographic models** based on perturbative QFT that describe a universe that started in a non-geometric strongly coupled phase.

- A class of such models based on a super-renormalizable QFT was custom-fit to data and shown to provide a competitive model to $\Lambda$CDM.

- Data from the Planck satellite should permit a definitive test of this holographic scenario.