Superamplitudes in $\mathcal{N} < 4$ SYM theory

(arXiv:1102.4348 Henriette Elvang, Yu-tin Huang and CP)

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Motivation

1. What is superamplitude?
   Encodes all “physical” scattering amplitudes that are related by SUSY as one Grassmann polynomial.

2. Why superamplitudes?
   - On-shell method, efficient computing
   - New symmetries, such as dual conformal symmetry and Yangian symmetry

3. Why $\mathcal{N} < 4$?
   Less symmetry, explore similarities and differences from $\mathcal{N} = 4$ SYM.
Tree level: Systematic truncation

- $\mathcal{N} = 4$ Pure SYM:
  
  $\Phi_4 = G^+ + \eta_a \lambda^a - \frac{1}{2!} \eta_a \eta_b S^{ab} - \frac{1}{3} \eta_a \eta_b \eta_c \lambda^{abc} + \eta_1 \eta_2 \eta_3 \eta_4 G^-$

  $\Phi_4$ represents a superfield containing a combination of the leading terms in the superpotential.

- Superamplitude: $\mathcal{F}_{n,\text{MHV}} \equiv \delta^{(8)} \left( \sum |k\rangle \eta_k \right) \langle 12 \rangle \langle 23 \rangle \cdots \langle n \rangle = \frac{4}{2^4} \left( \sum_{i,j=1}^n \langle ij \rangle \eta_{ia} \eta_{ja} \right) \prod_{a=1}^{\mathcal{N}} \langle 12 \rangle \langle 23 \rangle \cdots \langle n \rangle$.

- Truncation to $\mathcal{N} = 1$:
  
  $\Phi_1 = G^+ + \eta \lambda^+$, (Positive helicity, setting $\eta_{2,3,4} \to 0$)

  $\Psi_1 = \eta G^- + \lambda^-$, (Negative helicity, integrating over $\eta_{2,3,4}$)

  $\mathcal{F}_{n,i,j,\text{MHV}} = (-1)^{\frac{1}{2} \mathcal{N} (\mathcal{N} - 1)} \left( \sum |k\rangle \eta_k \right) \langle 12 \rangle \langle 23 \rangle \cdots \langle n \rangle$
Tree level: Systematic truncation

- **Super-BCFW recursion relation:**
  \[ \left[ I, J \right] \text{-shift: } \left| \hat{I} \right\rangle = \left| I \right\rangle + z \left| J \right\rangle, \quad \left| \hat{J} \right\rangle = \left| J \right\rangle - z \left| I \right\rangle, \quad \hat{\eta}_I = \eta_I + z \eta_J \]

  \[ \mathcal{F} = \sum_i \frac{\hat{F}_L^i \hat{F}_R^i}{P_i^2} \]

  "Good shift": the shifted amplitudes falloff as \( \frac{1}{z} \) (or better) when \( z \to \infty \).

  All shifts are "good" in \( \mathcal{N} = 4 \) SYM, while \([\Phi, \Psi]\) shift is "bad" when \( \mathcal{N} \leq 2 \).

**Key fact**

\([\Phi, \Psi]\) shift is "bad" when \( \mathcal{N} \leq 2 \)
Loop level: NOT a simple truncation

\[ A^{1-\text{loop}} = \sum_i C^i_{\text{box}} I^i_{\text{box}} + \sum_i C^i_{\text{triangle}} I^i_{\text{triangle}} + \sum_i C^i_{\text{bubble}} I^i_{\text{bubble}} + R. \]

- Interested in logarithmic UV divergence \( \Rightarrow I^i_{\text{bubble}} \) only.

Upshot: UV divergence at 1-loop is closely related to the BCFW shift at tree level.
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\[ \Phi \quad \Psi \]
\[ L \quad R \quad L \quad R \]
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- \( C^i_{\text{bubble}} \) comes from residual at \( z \to \infty \) of BCFW \([\ell_2, \ell_1]\)-shift.

- In dim-reg, \( \sum_i C^i_{\text{bubble}} = -b_0 A_{\text{tree}} \quad (\beta = -\frac{b_0}{(4\pi)^2} g^3 + \ldots) \)

**Upshot**

UV divergence at 1-loop is closely related to the BCFW shift divergence at tree level.
Interplay between 6D and 4D

- The 4D amplitudes can be obtained from the 6D amplitudes:
  1. 6D with \( P_4 = P_5 = 0 \) \( \Rightarrow \) 4D massless amplitudes.
  2. 6D with \( P_4, P_5 \neq 0 \) \( \Rightarrow \) 4D massive amplitudes.

- Mapping after reduction:
  \[ \begin{align*}
  D = 4, \mathcal{N} = 4 & \quad \text{truncation} \\
  D = 4, \mathcal{N} = 2 & \quad \text{Half Fourier transform:} (\eta_2, \eta_3) \quad \Rightarrow \quad D = 6, \mathcal{N} = (1, 1) \quad \text{truncation} \\
  D = 4, \mathcal{N} = 2 & \quad \text{Half Fourier transform:} (\eta_2) \quad \Rightarrow \quad D = 6, \mathcal{N} = (1, 0)
  \end{align*} \]

- 6D amplitudes have no “helicity sectors”: truncation of 6D amplitudes gives a mixture of \( \mathcal{N}^K \) MHV 4D amplitudes.

Lesson

6D amplitudes provide organizational guide for 4D amplitudes
Thank You