A Conformal Phase Transition in N=4 SYM Coupled to Defect Fermions

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Introduction

• In this talk, we will analyze a system of 2+1-dimensional interacting fermions with a tunable coupling, which undergoes a conformal phase transition (CPT) at coupling $\lambda_c$.

• Below criticality, the theory is conformal and fermions are massless; above it they get a dynamically generated mass exhibiting Berezinski-Kosterlitz-Thoules (BKT) scaling: $m \sim \Lambda \exp(-b/\sqrt{\lambda - \lambda_c})$. 

\[ \begin{align*}
\text{m} & \quad \lambda \\
\end{align*} \]
Some motivation

• Such a CPT is thought to occur at the phase transition as function of $N_f/N_c$ in QCD.

• Much recent work on studying these transitions using holography.

[Kaplan, Lee, Son, Stephanov (0905.4752); Jensen, Karch, Son, Thompson (1002.3159); Iqbal, Liu, Mezei, Si (1003.0010); etc]
N=4 SYM on a 2+1-dimensional defect

• Consider 4d N=4 SYM with gauge group SU(N) coupled to fermions on a codimension 1 defect

• Low-energy theory of N D3-branes intersecting a D7 brane in 2+1 dimensions:

\[
\begin{array}{cccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
D3 & x & x & x & x & & & & & \\
D7 & x & x & x & & x & x & x & x & x \\
\end{array}
\]

• Can be analyzed using QFT at weak coupling, gravity at strong coupling
Weak Coupling

• The field theory action is

\[ S = S_{N=4} + \int d^3x (i\bar{\psi}\gamma^\mu \psi + g\bar{\psi}A_\mu \psi + g\bar{\psi}\psi \phi^9) \]

• The object of interest is the self-energy \( \Sigma(p) \) of the fermion, which satisfies

\[ \frac{d}{dp} (p^2 \Sigma'(p)) + C(\lambda) \Sigma(p) = 0 \]

where \( C(\lambda) \) at weak coupling is

\[ C(\lambda) = \frac{\lambda}{\pi^3} + O(\lambda^2) \]
• One can write the fermion self energy as 

\[
\Sigma(p) = m \left( \frac{p}{\mu} \right)^\gamma + \frac{\langle \bar{\psi} \psi \rangle}{p} \left( \frac{\mu}{p} \right)^\gamma
\]

where the anomalous dimension \( \gamma \) is

\[
\gamma(\lambda) = -\frac{1}{2} + \sqrt{\frac{1}{4} - C(\lambda)}
\]

and a phase transition occurs when \( \kappa(\lambda) = C(\lambda) - \frac{1}{4} \) vanishes.

**Problem**: this happens when \( \lambda \) is order 1.
Strong Coupling

- Replace D3 branes with geometry:
  \[ ds^2 = \left( \frac{T}{L} \right)^2 dx_\mu dx^\mu + \left( \frac{L}{r} \right)^2 (d\rho^2 + \rho^2 d\Omega_3^2 + (dx^9)^2) \]

- \( f(\rho) \) parametrizes position of D7 brane in the x9 direction.

- DBI action of D7 brane:
  \[
  S_{D7} = \int d^3x \int_0^\Lambda d\rho \frac{L^2}{\rho^2 + f(\rho)^2} \rho^4 \sqrt{1 + f'(\rho)^2} \rightarrow \frac{\partial}{\partial \rho} (\rho^2 f'(\rho)) + 2f(\rho) + O(f^2) = 0 \quad \text{large } \rho
  \]

- Large \( \rho \) limit of EOM for \( f \) has same form as weak coupling equation for \( \Sigma \)

- Making this identification, \( \kappa \) at strong coupling = 7/4.
A Phase Transition?

\[
\kappa = -\frac{1}{4}
\]

\[
\lambda = \frac{7}{4}
\]
Dimensional Continuation

- Replace D7 brane with Dp:

\[
\begin{array}{cccccccccc}
D3 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
Dp & d & 4-d & n & 6-n & 6
\end{array}
\]

- Perturbative calculation now leads to

\[
\kappa = \frac{(d + 6 - n)\lambda}{8\pi} \frac{\Gamma(2 - \frac{d}{2})\Gamma(\frac{d-1}{2})}{\Gamma(\frac{d}{2})} - \left(\frac{d - 2}{2}\right)^2
\]

- Phase transition at weak coupling near \(d=2\).
Gravitational $\varepsilon$-expansion

- Alternatively, keep $d=3$ but push transition into gravity regime by tuning $n$. DBI action generalizes to

$$S = \int d^d x \int d\rho \left( \frac{\rho^2 + f^2}{L^2} \right)^{\frac{d-n}{2}} \rho^{n-1} \sqrt{1 + f'^2},$$

with phase transition at $\kappa_\infty = \lim_{\lambda \to \infty} \kappa(\lambda) = n - d - \left( \frac{d-2}{2} \right)^2$

- BKT limit: $n=13/4$, $k \to 0$, $\Lambda \to \infty$, $f(0)$ fixed
Observables

• We can now compute many interesting observables using gravity.

• Meson spectrum: expand around classical $f(\rho)$ to quadratic order. Lowest-lying mass in BKT limit is

$$m^2 \approx 0.44 f(0)^2$$

• Vector mesons, pions

• Finite temperature symmetry restoration

• Chemical potential, external fields, etc.
Summary

• We studied the physics of a conformal phase transition that occurs at coupling of order 1, by using dimensional continuation to push it to strong coupling

• Future directions: compute $\alpha'$ corrections; try to extend to QCD