

Extra Dimensions Summary/Review

Lecture 8: Spring 2009 Compton Lecture Series:

From Quantum Mechanics to the String

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- To make a dimension “small” you compactify it
 - Normally, zero dimensions is a point, one dimension is a line, two dimensions is a plane, three dimensions is a volume of space, and so on.
 - If you have one dimension, and you compactify it, you end up with a circle. In both cases position is described by a single number, but on a circle, if the circumference is length L , then the position $x = 0$ describes the same point as the position $x = L$.
 - If you are at distances much smaller than L , then the circle looks like a line (one non-compact dimension) If you are at distances much larger than L , then the circle looks like a point (zero non-compact dimensions).
 - We are used to thinking about a circle imbedded in a plane, but it’s possible for a line to be compact without reference to some larger dimensional system. In this case, the key feature is that if you travel far enough along the line, you come back to where you started. Although there are many figures you could draw in a plane that compactify a line, they are all exactly the same without reference to the plane: they all have this feature. There is only one way to compactify a line.
 - If you have two dimensions and you compactify one of them, then you have a cylinder. Very close up, the cylinder just looks like a plane. Far away, it looks like a line. Generally, if you have a compact dimension of size L , then at scales much smaller than L it looks like you have a non-compact dimension, and at scales much larger than L the compact dimension disappears altogether.
 - If we want to compactify two dimensions at once, there are many ways of doing this. (Sphere, torus, etc.) All of these are fundamentally different, even without thinking about how they look in a larger 3-space. With more than 2-dimensions you have even more options for distinct compactifications.
- The behavior of gravity changes depending on the number of spatial dimensions.
 - In 3-dimensions, the gravitational field is $g = \frac{G_3 M}{r^2}$.
 - We can represent the gravitational field by drawing field lines. The more densely spaced the field lines are, the stronger the gravitational field is.
 - If you draw a sphere around a gravitational source, the “flux” of the field through this sphere is a measure of how many field lines pass through it.
 - This doesn’t change if you make the sphere bigger or smaller. This is because though the field is getting weaker as you increase the distance from the source, the area of the sphere is also getting bigger. The surface area of the sphere is $A = 4\pi r^2$, so the flux of the field is $g \times A = 4\pi G_3 M$, which is independent of the distance from the source. This is a fundamental property of the gravitational field.
 - If you consider gravity in 2 spatial dimensions instead of 3, then the source should be surrounded by a circle instead of a sphere, and the flux of the field across the circle should be independent of how big you make the circle. Since the circumference of the circle is $2\pi r$, this tells us that the law of gravity should be $g = \frac{G_2 M}{r}$ in order to compensate properly.
 - Generalizing to d spatial dimensions, you surround a gravitational source with a $(d-1)$ -sphere, which has a “surface volume” that increases with radius like r^{d-1} . Thus, the law of gravity in a different number of dimensions should be $g = \frac{G_d M}{r^{d-1}}$.

- If some dimensions are compactified, the strength of gravity changes as you go from distances much larger than the scale of compactification to distance much smaller.
 - If we go back to 2 dimensions, one of which is compactified, and study the gravitational field lines, we can see that at distance much smaller than the circular size of the cylinder the field looks just the way it should on a plane. But as you zoom out, the field lines on the cylinder that don't go straight down it have nowhere to go, they bend back on themselves. As you go to very long distances, it just looks like one field line along the one non-compact dimension. It behaves like gravity field lines in 1 dimension.
 - Suppose we live in d dimensions, one of which is compact with size L , and there is a gravitational mass M and we are a distance r from it. For $r \gg L$ (when we're really far away compared to the size of the compact dimension) we feel a gravitational field $\frac{G_{d-1}M}{r^{d-2}}$. For $r \ll L$ (when we're really close compared to the size of the compact dimension) we feel a gravitational field $\frac{G_d M}{r^{d-1}}$. The gravitational field should vary smoothly between these two extremes. In order for this to happen we should have $G_{d-1}L = G_d$. Since Newton's constant controls the strength of the gravitational force, this tells us that at very short distance scales the strength of gravity could change depending on the size of compact dimensions.
 - Graphically, we can see that the change-over from the $\frac{G_{d-1}M}{r^{d-2}}$ behavior at long distances to the $\frac{G_d M}{r^{d-1}}$ behavior at short distances might make the gravitational field much stronger at short distances than we might otherwise have thought. This might mean, for example, that quantum gravity effects are important at significantly lower energies than we would otherwise think.
- With large compact extra dimensions, it might be possible to create black holes at the LHC.
 - The $1/r^2$ law for gravity is very well established at solar-system and human scales. At very short distance scales it is difficult to study because other forces are so much stronger. However, precision measurements have verified the $1/r^2$ law for gravity down to scales of about .1 mm. This means that extra dimensions, if they exist, must be smaller than this.
 - The horizon of a black hole in 3 spatial dimensions is $R = \frac{G_3 M}{c^2}$. If we imagine creating a black hole out of two colliding particles at the LHC, then M should be twice the mass of the proton, c is the known speed of light, and G_3 is the observed Newton's constant in 3 spatial dimensions. We can relate this radius to an energy required to get the protons this close to each other with $E = \frac{hc}{R}$. Plugging in the numbers gives us 5×10^{35} TeV as the required energy. The LHC only reaches 14 TeV.
 - However, the horizon of a black hole in 9 spatial dimensions is $R = \left(\frac{G_9 M}{c^2}\right)^{1/7} = \left(\frac{G_3 V_6 M}{c^2}\right)^{1/7}$ with V_6 the 6-volume of the extra compact dimensions.
 - If we assume the extra dimensions are only the size of 100 femtometers (10^{-13} meters), then the energy needed to produce black holes would be only 7.8 TeV, well within the capabilities of the LHC.
- Particles that move in extra, compact dimensions produce a Kaluza-Klein tower of particles of different masses.
 - Gravity is the only force that *must* interact with extra dimensions, if they exist. Other particles and forces might be confined to a lower dimensional surface inside the extra dimensions.
 - Suppose a particle does travel in 2-dimensions, one of which is compact. It is moving on a cylinder.
 - It's generic motion (without acceleration) will be a spiral down the cylinder. At long distance scales this will be observed as motion down the line.
 - The particle's motion in the compact direction does give the particle more energy, though. At long distance scales this energy will be experienced as a more massive particle traveling in one dimension.

- Because of this, one type of particle traveling in several dimensions, some of which are compact, produces many types of particles of many different masses, at long distance scales.
- Motion in the compact directions is quantized, which means that the possible masses of these particles form a discrete tower (called a Kaluza-Klein tower) of particle masses.
- The array of masses will depend on the shapes of the compact dimensions.
- If we do really live in a $9 + 1$ dimensional stringy world, then our every-day experience of four dimensional physics must depend strongly on the way the extra 6 spatial dimensions are wrapped up.