From Quantum Mechanics to String Theory

- Relativity (why it makes sense) ✓
- Quantum mechanics: measurements and uncertainty ✓
- Smashing things together: from Rutherford to the LHC ✓
- Particle Interactions ✓
- Quarks and the Strong Force ✓
- Symmetry and Unification ✓
- String Theory: a different kind of unification ✓
- Extra Dimensions
- Strings and the Strong Force
Gravity Summary

Gravity is different from the other forces because of the equivalence of gravitational and inertial mass. This leads to the curved space interpretation.

Black holes are a prediction of this. Hawking radiation from black holes seems to be irreversible, in conflict with QM.

Gravity interacts with the vacuum energy, Observation shows it is positive and small.

Gravity quantization breaks down at high energies in a way that is naturally solved by string theory.

Strings vibrating in a variety of ways give rise to particles of different masses and spins, including gravitons and photons.

Strings may solve the black hole and cosmological constant problems as well, but this is not yet clear.

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Extra Dimensions
Dimensions: The traditional geometric view

- Zero dimensions: a point
- One dimension: a line
- Two dimensions: a plane
- Three dimensions: a volume
- Etc... (hypervolumes)
- Our experience of the universe has 3 spatial dimensions and 1 time dimension
Compactification:
One Dimension

- A normal (non-compact) one dimensional system: a line

  \[ x = 0 \]

  Position on a line: a number \( x \) from \(-\infty\) to \(+\infty\)

- How do we make this small? A compact one dimensional system: a circle

  Position on a circle: a number \( x \) from 0 to \( L \), but \( x = 0 \) is the same position as \( x = L \)
Compactification: One Dimension

- \( L \) is the size of the compactified dimension

Suppose we are looking at distance scales that are much smaller than \( L \). It is as if we have “zoomed in” on the circle. This makes it look very much like a line.

If instead we are looking at distance scales that are much larger than \( L \) it is as if we have “zoomed out” on the circle. Now it looks like a point.
Compactification: One Dimension

When we picture a circle, we picture it inside a plane. The curved 1-dimensional object is imbedded inside a 2-dimensional object.

This is not actually necessary--it is possible to define a circle without reference to a second dimension. It is a 1-dimensional object that, if you move along it a distance L, you get back to where you started.

In a plane, it is possible to draw many figures that “compactify” one dimension.

Without the plane, they are all the same: along them you come back to where you started. There is really only way to compactify one dimension.
Compactification:

One Dimension

Suppose we have 2-dimensions, with one of them compactified.

The world is like the surface of a cylinder.

Close up, this looks like a plane. At large distance scales, it looks like a line.

General Rule: if a compactified dimension has size L, then at scales much smaller than L it looks like the dimension is not compact. At scales much larger than L it looks like the dimension disappears altogether.

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Compactification:
2 dimension and more

If we want to compactify 2 dimensions at once, there is more than one way to do it.

Sphere, torus, and so on. All are fundamentally different (without reference to external space). An infinite number of them, labeled by the number of “handles”.

More than 2-dimensions can be compactified in even more ways. (Describing these types of surfaces is the subject of topology)
Gravity

Newton’s Law for force between two masses:

\[ F = \frac{GM_1 M_2}{r^2} \]

Gravitational field about a mass \( M \):

\[ g = \frac{GM}{r^2} \]

Field line picture: represent stronger field strength by denser field lines

Imagine a sphere around the source, radius \( r \). How many field lines pass through the sphere? (this is the flux of the gravitational field)
Gravitational Flux

- the “flux” (the number of field lines passing through the sphere) doesn’t change, no matter how big we make the sphere.

- The field gets weaker, but the area of the sphere gets bigger.

- Area of sphere: \( A = 4\pi r^2 \)

- Strength of field: \( g = \frac{GM}{r^2} \)

- Flux of field: \( A \times g = 4\pi r^2 \times \frac{GM}{r^2} = 4\pi GM \)

- This is an essential property of the gravitational field.
Gravity in 2-dimensions

- In 3d the gravitational field source is surrounded by a sphere. In 2d the field source is surrounded by a circle of radius $r$ instead.

- The flux through the circle should stay constant as we increase the radius $r$.

- The surface area of a sphere should be replaced by the circumference of a circle $2\pi r$.

- Need the gravitational field to decrease like $1/r$ to compensate.

- Gravitational field: $g_2 = \frac{G_2 M}{r}$.
Suppose we live in $d$ spatial dimensions and 1 time dimension.

What you call a "sphere", we call a 2-sphere (the set of all points equal distance from the center is a 2-dimensional surface).

In $d$ spatial dimensions the object we’re interested in is a $(d-1)$-sphere.

\[ S^1 : 2\pi r, \quad S^2 : 4\pi r^2, \quad S^3 : 2\pi^2 r^3, \quad S^4 : \frac{8\pi^2}{3} r^4, \ldots \]

The important feature is that the surface (hyper)-area of a $(d-1)$ sphere increases with radius $r$ at a rate $r^{d-1}$.

This means the gravitational field must behave as $g_d = \frac{G_d M}{r^{d-1}}$. 

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Gravity and Compactification

Gravity behaves differently in a different number of spatial dimensions.

What if one or more dimension is compactified? (Size L)

Example: 2 spatial dimensions, 1 compactified

At distances much smaller than L, the field looks like it would in 2 normal dimensions

\[ g \approx G_1 M \]

At distances much larger than L, the field looks like it would in 1 normal dimension

\[ g \approx \frac{G_2 M}{r} \]
Gravity and Compactification

Suppose we live in $d$ spatial dimensions, 1 of which is compact with a size $L$.

There is a gravitational source (a mass $M$) at some location. You are a distance $r$ away from it.

For $r \gg L$, you feel a gravitational field $\frac{G_{d-1}M}{r^{d-2}}$.

For $r \ll L$, you feel a gravitational field $\frac{G_d M}{r^{d-1}}$.

For the gravitational field to “match up” around $r = L$, we should have $\frac{G_{d-1}M}{L^{d-2}} = \frac{G_d M}{L^{d-1}}$, $G_{d-1}L = G_d$.

Newton’s constant determines the strength of the gravitational force. This tells us that as we go to distance scales shorter than $L$, the strength of gravity appears to change.
Gravity and Compactification

Graphically, we can see what happens to the gravitational field.

At longer distances, it looks like you have one rule for determining the strength of the gravitational field.

As you move to distance scales shorter than the size of the compactified dimension, the field begins to follow a different curve.

It grows strong much faster than you though it was going to.

Implication: we thought we would have to go all the way to the Planck scale to see quantized gravity. If there are extra dimensions, this might happen sooner.
Gravity and Experiment

Gravity is pretty well understood from the distance scales of the solar system down to those of humans. Here, it behaves as if there are 3 spatial dimensions.

Short distance scales are difficult because gravity is so much weaker than the other forces.

Tabletop precision gravity experiments have explored distance scales down to about .1 mm, with no evidence of extra dimensions found.

Old thinking assumed that compactification would be at the Planck scale, but there’s no evidence it has to be.

We are looking for as many as 7 extra dimensions, not necessarily compactified at the same distance scales.
Black Holes at the LHC

Assuming 3 spatial dimensions, the horizon of a black hole is

\[ R = \frac{G_3M}{c^2} \]

To make a black hole of 2 protons requires energy of

\[ E = \frac{hc}{R} = \frac{hc^3}{G_3M} = 5 \times 10^{35} \text{ TeV} \]

Suppose there are 10 dimensions, and the six-volume of the compactified dimensions is \( V_6 \). The horizon of a black hole in 10 dimensions is

\[ R = \left( \frac{G_9M}{c^2} \right)^{1/7} = \left( \frac{V_6G_3M}{c^2} \right)^{1/7} \]

If the compactified dimensions are on the scale of 100 femto-meters, the required energy would be

\[ E = \frac{hc}{R} = hc \left( \frac{c^2}{V_6G_3M} \right)^{1/7} = 7.8 \text{ TeV} \]
Why Gravity?

We have discussed the behavior of gravity in extra dimensions, why not other fields?

Can't the same things be said for an electromagnetic field, for example?

It's possible to imagine a scenario where we live on a 3-dimensional membrane inside a higher dimensional world, and some forces act only inside the membrane (so they are completely 3-dimensional).

Gravity is the curvature of space-time, it's not possible to confine it to the membrane. If there are extra dimensions, then gravity is the one force that must be effected by this
Particle Motion and Extra Dimensions

Suppose there's a particle moving on a 2-dimensional surface, with one dimension compactified.

On this surface, traveling in "a straight line" is (generically) moving in a spiral down the cylinder.

But at long distances, where the two dimensions become one, it just looks like the particle is traveling down the line.
Kaluza-Klein Towers

The particle may be traveling very fast but have most of it's motion in the compact direction (a tight spiral). This particle may have a great deal of energy though from far away it doesn't appear to be moving very fast.

The energy from motion in the compact direction becomes mass.

One type of particle traveling on the cylinder will look like different particles with different masses, traveling on the line.

The amount of momentum in the compact dimension is quantized, creating a discrete tower of particles of increasing mass, called a Kaluza-Klein tower.

The array of masses will depend on the shapes of the compactified dimensions.
Extra Dimensions Summary

- One compact dimension is a circle. Compactifying more than 1 dimension can happen in a variety of ways. At long distances the compactified dimensions disappear.

- In 3-dimensions, the force of gravity behaves as $1/r^2$. In $d$ dimensions, it decreases as $1/r^{(d-1)}$.

- With compactified dimensions, the strength of gravity changes as you go to shorter distances scales.

- If there are large enough compactified dimensions, this could lead to black holes at the LHC.

- If a particle moves in several dimensions, of which some are compact, then the particle's motion creates a Kaluza-Klein tower of particles of different masses.