Quantum Mechanics Summary/Review

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From Quantum Mechanics to the String

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- For a given observable quantity (position, momentum, energy, spin) particles generically exist in a superposition of possible values
  - The smeared out possible positions of a particle form a wave.
  - The wavelength of this wave λ is a measure of how smeared out the particle is. An estimate for this wavelength is
    $$\frac{h}{mv}$$
    where m is the mass of the particle, v is the velocity, and h is a constant of nature known as "Planck's constant."
  - The larger λ is, the more like a wave the object behaves. The smaller λ is, the more like a particle it behaves.
- **Measuring a quantity changes the state of the particle**, forcing it into a state of definite value for this quantity
  - Measuring the position of the particle collapses the wave into a "spike" of definite position.
  - The "height" of the wave at a given position before measurement is related to the probability that the particle will be found at that position.
  - Over time, the spike will generally spread out again, and the information about position will be lost.
- Many quantities (for example position and momentum) can't be known simultaneously
  - The spread of the wave can also be thought of as the uncertainty in position. Similarly, there is some uncertainty in the momentum of a particle
  - These uncertainties satisfy an inequality,
    $$\Delta x \Delta p \geq \frac{h}{4\pi},$$
    which expresses the fact that the more accurately you know the position of a particle, the less accurately you know the momentum.
  - In an actual laboratory, if you try to measure both the position and momentum of a particle: first you measure the position, after which the particle is in a state of definite position but you don't know it's momentum. Then you measure the momentum, after which the particle has definite momentum. But this measurement has actually changed the state of the particle, and the information you had about the position of the particle is no longer accurate. You now know momentum, but not position.
- Localizing a particle in a spatial region gives it quantized energy values
  - Example: consider a particle in one dimension confined to a box.
  - The "wave" of positions must fit into the box evenly, this means there is a discrete set of possible states.

  - These are states of definite energy; the more "wiggles" the wave has, the higher energy the state is.
  - The state with the longest wave is the "ground state." any particle in the box must have at least as much energy as a particle in the ground state.
  - In general, the quantization of energy states for a particle comes from confining it to a limited spatial region. The typical example of this in particle physics is when one particle is held close to another by some force. For example, an electron bound to a nucleus by the electromagnetic force has discrete energy levels.

- Particles possess intrinsic, quantized angular momentum known as "spin"
  - This angular momentum, together with the charge of a particle creates a magnetic field around the particle and interacts with external magnetic fields. (This is the cause of natural magnets.)
  - The total amount of spin for a particle is fixed by the type of particle: electrons always have "spin 1/2", photons always have "spin 1", and so on.
  - The components of spin in each spatial dimension take quantized values; for a spin 1/2 particle the spin in a given direction is either $+$ or $-$.
  - The three spatial components of spin for a particle also satisfy an uncertainty principle: you can one know one component at a time.

- Planck's constant determines the scale at which quantum mechanical effects become important
  - If you could change the value of Planck's constant, and set it equal to zero, you could get rid of quantum mechanical effects
  - The "wavelength" of particles given by $\frac{h}{mv}$ would all be zero: their position waves would be infinitely peaked at a specific value, giving them definite position.
  - The uncertainty principle $\Delta x \Delta p \geq \frac{h}{4\pi}$ would disappear, allowing us to know all quantities infinitely well simultaneously.
  - The spaces between quantized energy levels would disappear, and they would become a continuum of energy levels.
  - We can't actually change the value of h. But at large (human-sized) scales where h is comparatively a very small number, we can mostly ignore quantum mechanical effects. It's the size of an object compared to h which determines how "quantum mechanical" it's behavior is.
  - Most of particle physics occurs in the region where both special relativity and quantum mechanics are important.