From Quantum Mechanics to String Theory

- Relativity (why it makes sense)
- Quantum mechanics: measurements and uncertainty
- Smashing things together: from Rutherford to the LHC
- Particle Interactions
- Quarks and the Strong Force
- Symmetry and Unification
- String Theory: a different kind of unification
- Extra Dimensions
- Strings and the Strong Force
Review of Relativity

- The laws of physics, and the speed of light, are the same for all inertial (moving with constant velocity) observers.

- This causes lengths and times to change depending on reference frame.

- Any physical observable (the result of an experiment) must be the same in all reference frames.

- Reference frame changes in space-time are analogous to rotations in space.

- No object can travel faster than the speed of light.

- Mass is a form of energy. The mass of an object can be defined as its energy/c^2 when at rest and isolated from external forces.
Quantum Mechanics:
Measurement and Uncertainty
Puzzle: The Stern-Gerlach Experiment

If you subject a beam of electrons to a magnetic field, the beam will split into two separate beams.

If you could slow the process down, you would see individual electrons deflected either up or down, with equal probability.

Nothing in classical physics can explain this.

Need a new theoretical framework.
Consider a quantity you want to measure for a system (example: the position of a particle)

**Classical understanding:** the particle has some definite position, and you determine what this position is when you measure it.

**Quantum mechanics understanding:** the particle exists in a state without definite position: it is “smeared out” in a wave of many possible positions

**The wave can travel:** this means the particle is traveling

[\lambda] is the width of the wave (usually measured where the wave is half its peak height)

The larger \( \lambda \) is, the more like a wave it behaves; the smaller \( \lambda \) is, the more like a particle

\[
\lambda \approx \frac{h}{mv}
\]

\( m \) is the mass of the particle, \( v \) the velocity, \( h \) a new constant: Planck’s constant

\[
h = 6.626 \times 10^{-34} \frac{m^2 kg}{s}
\]
Performing a measurement on a system changes the state of the system.

Measuring position “collapses” the wave into a spike of definite position.

The “height” of the wave at a given location before measurement gives the probability that the measurement will yield this value of location.

Over time, the spike will often spread out again into a wave similar to the one it started with (so you will lose information about position).
Uncertainty

λ is also the uncertainty in position Δx. Most states have some uncertainty in position, though states of definite position do exist.

We might also measure momentum \( p = \text{mass} \times \text{velocity} \): most states have uncertainty in momentum \( \Delta p \), some have definite momentum.

Some measurable quantities cannot be known to arbitrary accuracy simultaneously: momentum and position.

The more accurately you know position, the less accurately you know momentum (and vice versa).

\[
\Delta x \Delta p \geq \frac{\hbar}{4\pi}
\]

What happens if you try to measure both? If you measure position first, then your particle has definite position. But when you measure momentum, this changes the state of the particle: now it has definite momentum, but you no longer know the position.
Quantization and Localization:

Where does the “quantization” come into Quantum Mechanics?

Simple example: a particle in one dimension confined to a “box”, particle is a wave that fits into the box

There is a discrete set of possible waves with increasing energy

The longest wave is the ground state. Any particle must have at least this much energy.

If the particle was not confined inside the box, any wavelength would be possible and there would be a continuum of energy states

Generalization: a localized particle (e.g., an electron in orbit around a nucleus) has quantized energy states: localization = quantization

\[ E_n = \frac{n^2 \hbar^2}{8mL^2} \]

\( m \) = mass

\( L \) = box size

\( n = 1, 2, 3, \ldots \)
Spin

- Classical objects can spin about some axis; a spinning object tends to stay spinning: this is angular momentum.

- Angular momentum has magnitude and direction (what direction does the axis of rotation point in).

- Magnitude is determined by how fast the object is rotating, how massive it is, and how this mass is distributed.

- Can also think of angular momentum as three components: one in each spatial direction.

- A charged object with angular momentum produces a magnetic field and is influenced by an external magnetic field.
Electrons behave as if they are spinning (e.g. they interact with magnetic fields)

The spinning of individual electrons in the atoms produces natural magnetism

But to generate the observed angular momentum classically, they would have to be spinning faster than the speed of light

This spin is quantum mechanical
Quantum Spin

Particles of a specific type will always have the same quantized magnitude angular momentum, labeled by positive half-integers.

The direction of the angular momentum is not fixed by particle type.

Think in terms of x-, y-, and z-components of spin $S_x, S_y, S_z$.

Measurements of $S_z$ (or $S_y, S_z$) are quantized: for spin 1/2 particles there are 2 possibilities: (+,-)

<table>
<thead>
<tr>
<th>Spin 1/2</th>
<th>electrons, protons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spin 1</td>
<td>photons</td>
</tr>
<tr>
<td>Spin 2</td>
<td>gravitons</td>
</tr>
<tr>
<td>Spin 0</td>
<td>Higgs</td>
</tr>
</tbody>
</table>

$S_x, S_y, S_z$ satisfy uncertainty relationships: you can only know one component of spin at a time.
Electrons are affected by the magnetic field because of their angular momentum.

The direction of deflection depends on the component of spin along the magnetic field (call it $S_z$).

Electrons initially have no definite $S_z$: exist in a quantum superposition of possible values.

The magnetic field performs a measurement. The electrons are forced into the $S_z = (+)$, or the $S_z = (-)$ state with equal probability.
Extending Stern-Gerlach

If apply a second magnetic field (pointing in the same direction) to one of the two beams, the result will be only one beam.

The beam coming into the second field has definite spin in the z-direction.
Suppose instead the second magnetic field is in the \textit{x-direction}, which will measure spin in the \textit{x}-direction.

The beam coming into the second field has definite spin in the \textit{z}-direction, but indefinite spin in the \textit{z}-direction.

The second field splits this into two beams, each of which has definite spin in the \textit{x}-direction.
Now if we attach a third field (again in the \textit{z-direction}) we are measuring the spin in the z-direction of particles with definite spin in the x-direction. \( S_x \) and \( S_z \) are quantities that can't be known at the same time. States of definite \( S_x \) cannot have definite \( S_z \), so we again get two beams after the third field.

This result would hold even if we re-mixed the two beams after the second field. \textit{It is the act of measuring \( S_x \) which has changed the particles, destroying our earlier information about \( S_z \).}
Planck’s Constant

The scale of quantum mechanics

Quantum mechanical effects are governed by the value of $h$ (Planck’s constant) the same way relativistic effects are governed by $c$ (the speed of light)

Suppose we could stand outside the universe with godlike powers and change the value of $h$. If we could set $h = 0$, we could “turn off” quantum mechanics

$$\lambda \approx \frac{h}{mv}$$

The wavelengths of massive particles would be zero, giving them all definite positions.

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

The uncertainty relation would go away, allowing precise simultaneous knowledge of all measurable quantities

$$E_n = \frac{n^2 h^2}{8 mL^2}$$

Quantization would smooth out into continuous ranges of (e.g.) energy

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Planck’s Constant

- Of course, we can’t actually change $h$

- But the scale of a quantity (size, angular momentum, energy) compared to $h$ will tell us if quantum mechanical effects are important, the same way the speed of an object compared to $c$ determines if relativistic effects are important.

- Most of particle physics occurs in the region where both are important.
Review of Quantum Mechanics

- In general, particles don’t exist in state of definite (position, momentum, energy, spin, etc.)
- Measuring one of these quantities forces the particle into a state of definite value
- Many quantities (position and momentum, different components of spin) can’t be known simultaneously
- Localizing a particle in a spatial region gives it quantized energy levels
- Particles possess intrinsic, quantized angular momentum
- Planck’s constant determines the scale where quantum mechanical effects become important