1. Consider a particle of mass $m$ moving in an infinite square well

$$V(x) = \begin{cases} 
0 & -\frac{a}{2} < x < \frac{a}{2} \\
\infty & x < -\frac{a}{2} \text{ and } x > \frac{a}{2}
\end{cases}$$

Now add a small perturbation to the potential

$$V'(x) = \begin{cases} 
-\lambda v & -\frac{a}{2} < x < 0 \\
\lambda v & 0 < x < \frac{a}{2}
\end{cases},$$

where $\lambda$ is a small number. Treating $V'(x)$ as a small perturbation, use perturbation theory to find:

(a) Energy eigenvalues to the second order of $\lambda$.

(b) Eigenstates to the first order of $\lambda$.

2. Consider a particle moving in a simple harmonic oscillator potential. The Hamiltonian is

$$H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2.$$ 

In addition, there is a small perturbation in the system

$$H' = \lambda x^3.$$ 

Use perturbation theory to find:

(a) Energy eigenvalues to the second order of $\lambda$.

(b) Eigenstates to the first order of $\lambda$.

3. Consider a system with the following Hamiltonian $H = H_0 + H_1$, where

$$H_0 = \begin{pmatrix} 
a_1 & 0 & 0 \\
0 & a_2 & 0 \\
0 & 0 & a_3
\end{pmatrix}, \quad H_1 = \lambda \times \begin{pmatrix} 
0 & 0 & b \\
0 & 0 & c \\
b & c & 0
\end{pmatrix}.$$

(a) Use perturbation theory to find energy eigenvalues to the second order of $\lambda$ and eigenstates to the first order of $\lambda$.

(b) Consider the special case of $c = 0$. Diagonalize the Hamiltonian exactly to find eigenvalues and eigenstates. Compare these with the results from using perturbation theory.