1. “Moose” model. Consider gauge symmetry $U(1)_1 \times U(1)_2$, with gauge coupling strengths $g_1$ and $g_2$, respectively. We also have a complex scalar, $\phi$. It has charge +1 under $U(1)_1$, and charge +1 under $U(1)_2$. The scalar potential of $\phi$ is

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2.$$ 

We assume $\mu^2 < 0$.

(a) Compute the masses of the gauge bosons, and find the mass eigenstates in terms of $U(1)_1 \times U(1)_2$ gauge bosons $A_1^\mu$ and $A_2^\mu$.

(b) What is the mass of the remaining scalar particle, the "Higgs boson" $h$?

(c) Write down the couplings between the Higgs boson and the gauge bosons. Use mass eigenstates of the gauge bosons.

(d) Suppose we have two type of Dirac fermions ("electrons"), $e_1$ and $e_2$. $e_1$ is charged under $U(1)_1$ with charge $-1$, and it is neutral under $U(1)_2$. On the other hand, $e_2$ is charged under $U(1)_2$ with charge $-1$, and it is neutral under $U(1)_1$.

Write the Lagrangian for the fermions, including all renormalizable couplings between fermions and gauge bosons (again use the mass eigenstates), and between the fermions and $\phi$. 