1. Prove the following relations for 2-component fermions.

(a) \((\xi^\dagger \bar{\sigma} \eta)^\dagger = \eta^\dagger \bar{\sigma} \xi\)

(b) \(\xi \sigma^\mu \eta^\dagger = -\eta^\dagger \bar{\sigma} \xi\)

(c) Using \(\sigma^\mu_{\alpha \dot{\alpha}} \bar{\sigma} \beta \beta = 2\delta^\beta_\alpha \delta^\dot{\beta}_{\dot{\alpha}}\) to prove \((\eta_1 \sigma^\mu_1) (\eta_2^\dagger \bar{\sigma} \mu_4) = -2(\eta_1 \eta_4) (\eta_2^\dagger \eta_3)\)

2. Consider the following theory with two species of Dirac fermions (4-component), and one scalar field.

\[
\mathcal{L} = \frac{1}{2} i \bar{\psi}_1 \gamma_\mu \psi_1 - \frac{1}{2} m_1 \bar{\psi}_1 \psi_1 + \frac{1}{2} i \bar{\psi}_2 \gamma_\mu \psi_2 - \frac{1}{2} m_2 \bar{\psi}_2 \psi_2 \\
+ \frac{1}{2} (\partial \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 \\
- g_1 \phi \bar{\psi}_1 \psi_1 - g_2 \phi \bar{\psi}_2 \psi_2
\]

(a) Write the scattering amplitude for

i. \(\psi_1 \psi_1 \rightarrow \phi \phi\),

ii. \(\bar{\psi}_1 \bar{\psi}_1 \rightarrow \phi \phi\),

iii. \(\psi_1 \psi_1 \rightarrow \psi_2 \psi_2\),

iv. \(\bar{\psi}_1 \bar{\psi}_1 \rightarrow \psi_2 \psi_2\),

v. \(\psi_1 \bar{\psi}_1 \rightarrow \psi_2 \bar{\psi}_2\),

where we have denoted particle by \(\psi_i\) and anti-particle by \(\bar{\psi}_i\).

(b) Write the one-loop amplitude for \(\psi_1 \bar{\psi}_1 \rightarrow \phi \phi\). Carry out the loop integral.