The Lagrangian is given as

$$\mathcal{L} = \frac{1}{2} \left[ (\partial \phi_1)^2 + (\partial \phi_2)^2 \right] - \frac{\lambda}{4!} (\phi_1^4 \phi_2^4) - \frac{2r}{4!} \phi_1^2 \phi_2^2$$  \hspace{1cm} (1)

For massless scalar fields, we write the renormalization conditions given by Eqn. (12.30) of P&S for both $\phi_1$ and $\phi_2$. For the $\rho$ coupling, the final diagram is replicated as:

Figure 1: Renormalization condition for the $\rho$ coupling.

The procedure for computing $\delta Z$, $\delta \lambda$, and $\delta \rho$ was outlined in section 10.2. From Eqn. (10.29), $\delta Z = 0$ at $O(\lambda, \rho)$, and receives only two-loop corrections. $\delta \lambda$ and $\delta \rho$ are

$$\delta \lambda = \frac{3}{32 \pi^2} \left( \lambda^2 + \frac{\rho^2}{9} \right) \log \left( \frac{\Lambda^2}{M^2} \right),$$  \hspace{1cm} (2)

$$\delta \rho = \frac{1}{16 \pi^2} \left( \lambda \rho + \frac{2}{3} \rho^2 \right) \log \left( \frac{\Lambda^2}{M^2} \right).$$  \hspace{1cm} (3)

See Eqns. (12.44-12.45).
c., d.

Since there are no $\mathcal{O}(\lambda, \rho)$ (one-loop) corrections to the propagator $G^{(2)}$, the $\gamma$ function is zero at this order (see P&S p. 413). From Eqn. (12.53), the $\beta$ functions and RGE’s are:

\[
\frac{\partial \lambda}{\partial \log M} = \beta_\lambda = -\frac{3}{16\pi^2} \left( \lambda^2 + \frac{\rho^2}{9} \right) \tag{4}
\]
\[
\frac{\partial \rho}{\partial \log M} = \beta_\rho = -\frac{\rho}{16\pi^2} \left( \lambda + \frac{2}{3} \rho \right) \tag{5}
\]

e.

Consider the $\beta$ function of the ratio $x = \rho/\lambda$:

\[
\beta_x = \frac{1}{\lambda^2} (\lambda \beta_\rho - \rho \beta_\lambda) = -\frac{\lambda x}{48\pi^2} (x - 3)(x - 1). \tag{6}
\]

There are fixed points at $x = 1, 3$. If $0 < x < 1$ at $M$, then $\beta_x$ is negative, and $x$ behaves as if it is asymptotically free, i.e. $x$ grows with decreasing energy or longer distance scales as it approaches the fixed point $x = 1$. If $1 < x < 3$ at $M$, then $\beta_x$ is negative, and we have QED-like behaviour for $x$, i.e. $x$ decreases with decreasing energy or longer distance scales as it approaches the fixed point $x = 1$.

f.

Using Eqn. (12.131), we find

\[
\beta^{(d)}_\lambda = -\epsilon \lambda + \beta^{(4)}_\lambda, \tag{7}
\]
\[
\beta^{(d)}_\rho = -\epsilon \rho + \beta^{(4)}_\rho, \tag{8}
\]

where $\beta^{(4)}$ are the $\beta$ functions computed in (c.)