We consider a system of 2 spin-1/2 particles. We have defined operators acting on the two spin-1/2 particles:

\[ s_1^x, s_1^y, s_1^z, s_1^\pm = s_1^x \pm is_1^y; \quad s_2^x, s_2^y, s_2^z, s_2^\pm = s_2^x \pm is_2^y \]

\[ \vec{S} = \vec{s}_1 + \vec{s}_2, \quad S^2 = (\vec{s}_1 + \vec{s}_2)^2 \]

\[ S_x = s_1^x + s_2^x, \quad S_y = s_1^y + s_2^y, \quad S_z = s_1^z + s_2^z, \quad S_\pm = s_1^\pm + s_2^\pm \]

1. Consider the following Hamiltonians

\[ H_1 = a s_1^z + b s_2^z, \quad H_2 = c \vec{s}_1 \cdot \vec{s}_2, \quad H_3 = a s_1^z + c \vec{s}_1 \cdot \vec{s}_2, \quad H_4 = a s_1^z + b s_2^z + c \vec{s}_1 \cdot \vec{s}_2 \]
\[ H_5 = a s_1^x + b s_2^x, \quad H_6 = a (s_1^x + s_1^y) \]

What are the conserved quantities for each of these Hamiltonians?

2. Considering the following Hamiltonian

\[ H = a \vec{s}_1 \cdot \vec{B} + c \vec{s}_1 \cdot \vec{s}_2 \]

Suppose the constant magnetic field is \( \vec{B} = B\hat{k} \).

(a) Find the eigenstates and eigenvalues of this Hamiltonian.

(b) Suppose at \( t = 0 \), particle 1 is polarized in the +x direction, and particle 2 is polarized in the +z direction. That is, the initial state of the system is \( |x+\rangle|z+\rangle \). Express this initial state in terms of eigenstate of the Hamiltonian.

(c) What is the state at a later time \( t \)?

(d) What is the expectation value of \( s_{1x} \) as a function of \( t \)?

(e) Suppose we measure \( \vec{s}_1 \cdot \vec{s}_2 \) at time \( t \), what are the possible outcomes?

(f) What are the probabilities of obtaining these outcomes?