Physics 234: Quantum Mechanics

Homework 3.

1. Take the eigenstate of $\vec{\sigma} \cdot \vec{n} = \sin \theta \cos \phi \sigma_1 + \sin \theta \sin \phi \sigma_2 + \cos \theta \sigma_3$ with eigenvalue +1, denoted as $|n+\rangle$.

   (a) Suppose we perform a measurement of the observable $\sigma_1$. What are the possible outcomes? What is the state after the measurement obtaining each of those outcomes?

   (b) Compute the density matrix $\rho$ in the basis of eigenstates of $\sigma_3$.

   (c) Compute $\text{Tr}(\rho)$ and $\text{Tr}(\rho^2)$.

   (d) Compute the expectation value of $\sigma_1$ for the state $|n+\rangle$.

2. We again work in basis of the eigenstate of $\sigma_3$, denoted as $|+\rangle$ and $|-\rangle$. Suppose the Hamiltonian of a two state system is given by

   $H = \begin{pmatrix} a & c \\ c^* & a \end{pmatrix}$

   Suppose at $t = 0$, the system is prepared in the state $|+\rangle$.

   (a) Find the eigenvalues and eigenstates of the Hamiltonian.

   (b) Find the state of the system at arbitrary time $t$.

   (c) Suppose we then perform a measurement of the observable $\sigma_2$. What is the probability (as a function of time) of getting +1?

   (d) Compute the expectation value of $\sigma_2$ as a function of time.