Introduction to the Idea of Twisted SUSY Nonlinear Sigma Model

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ABSTRACT: This note is prepared for the journal club talk. Here we introduce the idea of dubbed topological field theories, especially the Witten type, and use the models raised in the Mirror manifold paper [1] as examples.

Contents

1 Topological Field Theories

This quarter the journal club focuses on topics related to topological field theories. Before we dive into it, we have to specify what do we mean by a topological field theory.
In many cases, the action for a quantum field theory defined over a generic manifold depends on the metric. Thus, intuitively we think the partition function depends on the metric endowed as a functional of the action. A topological quantum field theory, instead, depends merely on the topological properties of the manifold (and of the maps between the *spacetime* manifold and the target space.) In such case, the corresponding partition function usually becomes a topological invariant and can be evaluated using topological math machinery.
To be more precise, let us try to specify the definition of a topological field theory that we will introduce or discuss in this talk [2].

1. We have a set of fields X defined on a Riemannian manifold (Σ, g).
2. There exists a BRST like nilpotent operator Q. (Thus, the physical states are characterized by cohomology class.)
3. A Q-exact energy-stress tensor.

Also we have to assume the path integration measure is Q and g-independent.
In a moment we will be showing a theory defined this way has a g-independent partition function. For some classes of operators, the correlation functions are also metric independent. Before diving into that, in the literature people usually divide topological field theories

into *Witten* type and *Schwarz* type. Theories in both categories enjoy properties 1 to 3. The major difference lies in the form of action. Theories of Witten type often have seemingly metric dependent actions, which turn out to be Q-exact. Those theories may be massaged by adding other topological terms into them, while they remain topological. On the other hand, Theories of Schwarz type often start with a metric-independent action. The metric dependence, or the energy stress tensor, comes from gauge fixing and ghosts fields. The known Chern-Simons theory belongs to the latter category.

The rest of this note is organized as follows. First we review some facts from BRST symmetry and quantization in gauge theories. Following that, the implications from properties 1-3 and be derived. Next we introduce the supersymmetric sigma model and try to explain how it can become a topological field theory by *twisting*.

2 BRST invariance

In this section we also provide a brief review of BRST invariance. Readers may go to Ref. [3, 4] for more elaborating and insightful discussions.

According to my understanding, the BRST formalism was invited to make the gauge invariance more manifest even after gauge fixing. In Faddeev-Popov scheme, after gauge fixing, the total Lagrangian is no longer manifestly gauge invariant due to the gauge fixing terms.

To be concrete, we may look at the following Largangian

$$\begin{aligned} \mathscr{L} =& \mathscr{L}_{\mathrm{YM}} + \mathscr{L}_{\mathrm{GF}} \\ =& -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\psi}(\mathrm{i}\not{D} - m)\psi + \frac{\xi}{2}(B^a)^2 + B^a\partial^\mu A^a_\mu + \bar{c}^a(-\partial^\mu D^{ac}_\mu)c^c, \end{aligned} \tag{2.1}$$

where ξ is the gauge fixing parameter, B^a is a Hubbard-Stratonovich field imposing the gauge condition $\partial_\mu A^\mu = 0$, and c^as are ghosts.

We know the Yang-Mills Lagrangian itself is gauge invariant, yet those gauge fixing terms are not, yet one can see this Lagrangian is invariant under the following transformation.

$$\begin{aligned} \delta A^a_\mu &= \epsilon D^{ac}_\mu c^c \\ \delta\psi &= \mathrm{i}g\epsilon c^a t^a \psi \\ \delta c^a &= -\frac{1}{2}g\epsilon f^{abc}c^b c^c \\ \delta\bar{c}^a &= \epsilon B^a \\ \delta B^a &= 0, \end{aligned} \tag{2.2}$$

where ϵ is a Grassmann parameter. Following Noether's recipe we know there is a conserved charge Q that generates this transformation. For an operator $\mathscr{O}$,

$$\delta\mathscr{O} = -\mathrm{i}[\epsilon Q, \mathscr{O}] = -\mathrm{i}\epsilon\{Q, \mathscr{O}\}. \tag{2.3}$$

Note that $\{...,...\}$ doesn't merely denote an anti-commutator. It's a graded commutator. Roughly speaking, it's a commutator for Bosonic fields and an anti-commutator for Fermionic fields. The most important feature is that for any fields $\mathscr{O}$, we have $[Q^2, \mathscr{O}] = 0$. Schurs' Lemma implies Q^2 is either 0 or proportional to identity operator. Nonetheless, since it carries non-zero ghost number, it can only be a nilpotent operator.

$$Q^2 = 0. \tag{2.4}$$

Introducing such a new symmetry helps us write the total Lagrangian as

$$\mathscr{L} = \mathscr{L}_{\mathrm{YM}} + \delta\mathscr{O}, \tag{2.5}$$

where the first part is gauge invariant and the latter is Q exact. As a consequence we can separate redundant gauge degrees of freedom using the cohomology group.
Before we put a period to cease this short review, we note that not only Lagrangians but physical states are characterized by the cohomology groups. $Q|\Psi\rangle = 0$ modulo states of form $Q|...\rangle$, say physical states are Q closed but not Q exact.

3 Consequences of a BRST charge Q

In the story we will be considering topological field theories of Witten type, meaning that the Lagrangian can be written as

$$\mathscr{L} = -\mathrm{i}\{Q, V\} \tag{3.1}$$

for a field functional V and the energy stress tensor can be written as

$$T_{\mu\nu} = -\mathrm{i}\{Q, \Lambda_{\mu\nu}\}. \tag{3.2}$$

To some extent, these 2 are not independent assumptions since the energy stress tensor, with a metric g on Σ, is defined as

$$\delta_g \int_\Sigma \mathscr{L} = \frac{1}{2}\int_\Sigma \sqrt{g}\, \delta g^{\mu\nu}\, T_{\mu\nu}. \tag{3.3}$$

Note that the existence of Q is owing to twisting the original supersymmetry in the theory (though we have yet explained what does it mean by twisting.) Thus the path integral measure is assumed invariant under Q transformation.
With these condition, we argue that the theory is topological in the sense that the partition function

$$Z = \int (\mathscr{D}X)\, e^{-t\int_\Sigma \mathscr{L}(X)}. \tag{3.4}$$

is invariant under the variation with respect to metric over Σ and coupling t.
To this end, we want to first show for any operator $\mathscr{O}(X)$, the expectation value $\{Q, \mathscr{O}\}$ vanishes. Looking at $\langle \mathscr{O} \rangle$

$$\langle \mathscr{O} \rangle = \int (\mathscr{D}X)\, \mathscr{O}\, e^{-t\int_\Sigma \mathscr{L}(X)}. \tag{3.5}$$

Making use of the supersymmetry of Q, we consider a variation $\delta\mathscr{A} = \epsilon\{Q, \mathscr{A}\}$ for the integrands and measures in the partition function. The Lagrangian itself is already Q closed, together with the fact that integration measure is supersymmetric,

$$\langle \mathscr{O} \rangle = \int (\mathscr{D}X)\, (\mathscr{O} + \epsilon\{Q, \mathscr{O}\})\, e^{-t\int_\Sigma \mathscr{L}(X)}. \tag{3.6}$$

If this identity hold for arbitrary ϵ, we conclude that $\langle\{Q, \mathscr{O}\}\rangle = 0$. With this in mind, the above 2 assertions can be justified in straightforward manners. First we consider a variation of Z with respect to the metric g on Σ.

$$\begin{aligned} \delta_g Z =& \int (\mathscr{D}X)\, (-t\delta_g \int_\Sigma \mathscr{L})\, e^{-t\int_\Sigma \mathscr{L}(X)} \\ =&\mathrm{i}\frac{t}{2} \int (\mathscr{D}X)\, \int_\Sigma \sqrt{g}\, \delta g^{\mu\nu}\, \{Q, \Lambda_{\mu\nu}\}\, e^{-t\int_\Sigma \mathscr{L}(X)} = 0. \end{aligned} \tag{3.7}$$

The second one follows similarly,

$$\delta_t Z = \mathrm{i} \int (\mathscr{D}X)\delta t \int_\Sigma \{Q, V\}\, e^{-t\int_\Sigma \mathscr{L}(X)} = 0. \tag{3.8}$$

Now that the partition function is metric and coupling independent, a natural question is that what kind of correlation functions have the privilege to share these properties.
To answer this question, let us consider again

$$\langle \mathscr{O} \rangle = \int (\mathscr{D}X)\, \mathscr{O}\, e^{-t\int_\Sigma \mathscr{L}} = \int (\mathscr{D}X)\, \mathscr{O}\, e^{-S[X]}. \tag{3.9}$$

This time let us vary the expectation value with respect to metric g.

$$\delta_g \langle \mathscr{O} \rangle = \int (\mathscr{D}X) \Big(\delta_g \mathscr{O} - \mathscr{O} \delta_g S[X] \Big)\, e^{-t\int_\Sigma \mathscr{L}}. \tag{3.10}$$

We have known that if the total operator sandwiched is a Q-commutator, $\delta_g\langle \mathscr{O} \rangle = 0$. Thus, if $\delta_g \mathscr{O} = \{Q, \mathscr{A}\}$ and $\{Q, \mathscr{O}\} = 0$, the sandwiched part can be written as $\{Q, ...\}$ and as a consequence $\delta_g\langle \mathscr{O} \rangle = 0$. Note that the condition $\{Q, \mathscr{O}\} = 0$ implies we are looking for Q-closed operator. Nonetheless, if $\mathscr{O}$ is Q-exact, the case is trivial. Similar to seeking physical states, metric-independent correlation functions require knowing the cohomology of Q.

4 The Non-Linear Sigma Model

Here generally speaking, a non-linear sigma model can be described as a map from a manifold $x^\mu \in \Sigma$ to the other $\phi^I \in X$, $\Phi : \Sigma \to X$. The model is given by the action

$$\int_\Sigma \mathrm{d}^d x \, g_{IJ}(\phi)\partial_\mu \phi^I \partial^\mu \phi^J \tag{4.1}$$

with g_{IJ} as a function of ϕ^I playing the role of metric on X. A particularly interesting case is when $d = 2$, where ϕ^Is are dimensionless. For various kind of g_{IJ}, the theory is renormalizable. Besides, the model can then be considered as fields defined on string world sheet. As one may already recognize, the Polyakov action, in this sense, can be regarded as a kind of Sigma model.

Also very often in even spacetime dimensions we use complex coordinates. Up to measure normalization factor $\mathrm{d}^2 x \to \mathrm{d}^2 z$ (which is $-\mathrm{i}\mathrm{d}z \wedge \mathrm{d}\bar{z}$ is Witten's paper). As we may have seen in some CFTs context in 2 dimensions, the tensor indices $\mu\nu...$ can be written in terms of z and $\bar{z}$. We can then actually separate our tangent bundle $T\Sigma$ into 2 parts $V^z \frac{\partial}{\partial z} = V^z \partial \in T^+\Sigma$ and $V^{\bar{z}} \frac{\partial}{\partial \bar{z}} = V^{\bar{z}} \bar{\partial} \in T^-\Sigma$. Correspondingly, one forms can be separated into 2 canonical sectors, say, holomorphic and anti-holomorphic sectors, $\omega_z \, \mathrm{d}z$ and $\omega_{\bar{z}} \, \mathrm{d}\bar{z}$. For a general complexified tensor, one can define *helicity* in terms of the difference between numbers of z indices and $\bar{z}$ indices.

To proper define the models raised in the following sections, we had better also assume some properties for the target space X. The safest choose is saying X is a Calabi-Yau manifold.[1] Since it is complex, we can also complexify the tangent space TX as $T^+X \oplus T^-X$, which indicates that we can write ϕ^J in terms of ϕ^i and $\phi^{\bar{i}}$.

Next we are seeking the supersymmetric extension of this non-linear sigma model. To this end, we need more structures, say, as least, spin bundles, to properly define fermions. Using the helicity notion introduced, we can have 2 kinds of canonical 1 form sections, the holomorphic $(1,0)$ part K and the anti-holomorphic $(0,1)$ part $\bar{K}$. The fermions with $+$ and $-$ helicity can then be, respectively, considered as fields defined on the tensor bundle $K_\Sigma^{1/2} \otimes TX$ and $\bar{K}_\Sigma^{1/2} \otimes TX$. Thus there are 4 types of fermions depending on if they are holomorphic or anti-holomorphic and the helicity they own. They are the sections of the following bundles.

$$K^{1/2} \otimes \Phi^*(T^+X),\ K^{1/2} \otimes \Phi^*(T^-X),\ \bar{K}^{1/2} \otimes \Phi^*(T^+X),\ \bar{K}^{1/2} \otimes \Phi^*(T^-X). \tag{4.2}$$

After introducing these terminology, an action for $\mathcal{N} = 2$ supersymmetric sigma model defined in the case that X is Kähler can be written down, by either brutal force of superfield

[1] Such a strong assumption is to avoid the issue of anomaly that occurs in the model B defined in the following sections.

construction,

$$I = 2t \int_\Sigma \mathrm{d}^2 z \left(\frac{1}{2} g_{IJ} \partial_z \phi^I \partial_{\bar{z}} \phi^J + \mathrm{i} \psi_-^{\bar{i}} D_z \psi_-^i g_{\bar{i}i} + \mathrm{i} \psi_+^{\bar{i}} D_{\bar{z}} \psi_+^i g_{\bar{i}i} + R_{i\bar{i}j\bar{j}} \psi_+^i \psi_+^{\bar{i}} \psi_-^j \psi_-^{\bar{j}} \right). \quad (4.3)$$

The fermions ψ_+^i and $\psi_+^{\bar{i}}$ live in $K^{1/2} \otimes \Phi^*(T^+X)$ and $K^{1/2} \otimes \Phi^*(T^-X)$ respectively. Similarly, ψ_-^i and $\psi_-^{\bar{i}}$ live in $\bar{K}^{1/2} \otimes \Phi^*(T^+X)$ and $\bar{K}^{1/2} \otimes \Phi^*(T^-X)$. For later discussion, let us list the supersymmetry transformation as follows.

$$\begin{aligned}
\delta\phi^i &= \mathrm{i}\alpha_- \psi_+^i + \mathrm{i}\alpha_+ \psi_-^i \\
\delta\phi^{\bar{i}} &= \mathrm{i}\tilde{\alpha}_- \psi_+^{\bar{i}} + \mathrm{i}\tilde{\alpha}_+ \psi_-^{\bar{i}} \\
\delta\psi_+^i &= -\tilde{\alpha}_- \partial_z \phi^i - \mathrm{i}\alpha_+ \psi_-^j \Gamma^i_{jm} \psi_+^m \\
\delta\psi_+^{\bar{i}} &= -\alpha_- \partial_z \phi^{\bar{i}} - \mathrm{i}\tilde{\alpha}_+ \psi_-^{\bar{j}} \Gamma^{\bar{i}}_{\bar{j}\bar{m}} \psi_+^{\bar{m}} \\
\delta\psi_-^i &= -\tilde{\alpha}_+ \partial_{\bar{z}} \phi^i - \mathrm{i}\alpha_- \psi_+^j \Gamma^i_{jm} \psi_-^m \\
\delta\psi_-^{\bar{i}} &= -\alpha_+ \partial_{\bar{z}} \phi^{\bar{i}} - \mathrm{i}\tilde{\alpha}_- \psi_+^{\bar{j}} \Gamma^{\bar{i}}_{\bar{j}\bar{m}} \psi_+^{\bar{m}}.
\end{aligned} \quad (4.4)$$

This set of supersymmetry transformation is not globally defined since they are also sections. In the following section we will see the twisting technique gives some of those parameters global meaning.

5 Twisting the Model

In the first introductory section we have seen the essential features of topological field theories. The problem essentially is rephrased in terms of Q, or its existence. By what mean can I construct a Q?

Twisting the model (4.3) is a way, tho the trick seems (to the author) really magical.

The procedure can be summarized as follows.

1. A model:
   Take ψ_+^i and $\psi_-^{\bar{i}}$ to be the sections of $\Phi^*(T^+X)$ and $\Phi^*(T^-X)$ respectively, and correspondingly, take $\psi_+^{\bar{i}}$ and ψ_-^i to be $(1,0)$ form on $\Phi^*(T^-X)$ and $(0,1)$ form on $\Phi^*(T^+X)$. The form of the Lagrangian is essentially unchanged, yet the covariant derivatives need proper modification.
   After twisting, we have a anti-commuting scalar $\chi^i = \psi_+^i$ and $\chi^{\bar{i}} = \psi_-^{\bar{i}}$ and two 1-forms $\psi_+^{\bar{i}} \equiv \psi_z^{\bar{i}}$ and $\psi_-^i \equiv \psi_{\bar{z}}^i$.
   Consequently, α_- and $\tilde{\alpha}_+$ no longer need to be sections. They can be globally defined functions. Choosing $\alpha_+ = \tilde{\alpha}_- = 0$ and $\alpha_- = \tilde{\alpha}_+ =$ constant, we have a global

transformation inherited from (4.4).

$$\begin{aligned}
\delta\phi^I &= \mathrm{i}\alpha\chi^I \\
\delta\chi^I &= 0 \\
\delta\psi_z^{\bar{i}} &= -\alpha\partial_z\phi^{\bar{i}} - \mathrm{i}\alpha\chi^{\bar{j}}\Gamma^{\bar{i}}_{\bar{j}\bar{m}}\psi_z^{\bar{m}} \\
\delta\psi_{\bar{z}}^{i} &= -\alpha\partial_{\bar{z}}\phi^{i} - \mathrm{i}\alpha\chi^{j}\Gamma^{i}_{jm}\psi_{\bar{z}}^{m}.
\end{aligned} \tag{5.1}$$

In terms of new variables, the A-twisted action reads

$$I_A = 2t\int_\Sigma \mathrm{d}^2z \left(\frac{1}{2}g_{IJ}\partial_z\phi^I\partial_{\bar{z}}\phi^J + \mathrm{i}\psi_z^{\bar{i}}D_{\bar{z}}\chi^i g_{\bar{i}i} + \right.$$
$$\left. \mathrm{i}\psi_{\bar{z}}^{i}D_z\chi^{\bar{i}}g_{\bar{i}i} - R_{i\bar{i}j\bar{j}}\psi_{\bar{z}}^{i}\psi_z^{\bar{i}}\chi^j\chi^{\bar{j}}\right). \tag{5.2}$$

The punchline here is that the charge Q corresponding to (5.1) helps us to write I_A as

$$I_A = \mathrm{i}t\int_\Sigma \mathrm{d}^2z\,\{Q,V\} + t\int_\Sigma \mathrm{d}^2z\left(\partial_z\phi^i\partial_{\bar{z}}\phi^{\bar{j}}g_{i\bar{j}} - \partial_{\bar{z}}\phi^i\partial_z\phi^{\bar{j}}g_{i\bar{j}}\right), \tag{5.3}$$

where

$$V = g_{i\bar{j}}(\psi_z^{\bar{i}}\partial_{\bar{z}}\phi^j + \partial_z\phi^{\bar{i}}\psi_{\bar{z}}^{j}). \tag{5.4}$$

As it is argued in previous sections, the first term is topological. The section term can be regarded as a pullback of Kähler form, which depends merely on the its cohomology and the homotopy class of Φ.

2. B model:
Instead, we take both $\psi_\pm^{\bar{i}}$ to be sections of $\Phi^*(T^-X)$, ψ_+^i to be a section of $K\otimes\Phi^*(T^+X)$ and ψ_-^i to be a section of $\bar{K}\otimes\Phi^*(T^+X)$.
As for this model, $\psi_\pm^{\bar{i}}$ become anti-commuting scalars. Witten considered

$$\begin{aligned}
\eta^{\bar{i}} &= \psi_+^{\bar{i}} + \psi_-^{\bar{i}} \\
\theta_i &= g_{i\bar{i}}(\psi_+^{\bar{i}} - \psi_-^{\bar{i}})
\end{aligned} \tag{5.5}$$

and a new 1-form ρ with $\rho_z^i = \psi_+^i$ and $\rho_{\bar{z}}^i = \psi_-^i$.
In model B, $\alpha_\pm$ are sections while $\tilde{\alpha}_\pm$ can be globally chosen as α. Setting $\alpha_\pm = 0$, we have another set of supersymmetry transformations

$$\delta\phi^i = 0$$
$$\delta\phi^{\bar{i}} = \mathrm{i}\alpha\eta^{\bar{i}}$$
$$\delta\eta^{\bar{i}} = \delta\theta_i = 0 \tag{5.6}$$
$$\delta\rho^i = -\alpha\,\mathrm{d}\phi^i. \tag{5.7}$$

The action then reads

$$I_B = t\int_\Sigma \mathrm{d}^2 z\Big(g_{IJ}\partial_z\phi^I\partial_{\bar{z}}\phi^J + \mathrm{i}\eta^{\bar{i}}(D_z\rho^i_{\bar{z}} + D_{\bar{z}}\rho^i_z)g_{i\bar{i}}\Big)$$
$$\mathrm{i}\theta_i(D_{\bar{z}}\rho^i_z - D_z\rho^i_{\bar{z}}) + R_{i\bar{i}j\bar{j}}\rho^i_z\rho^j_{\bar{z}}\eta^{\bar{i}}\theta_k g^{k\bar{j}}\Big). \tag{5.8}$$

Similarly, this can be rewritten as

$$I_B = \mathrm{i}t\int\{Q,V\} + t\int_\Sigma\Big(-\theta_i D\rho^i - \frac{\mathrm{i}}{2}R_{i\bar{i}j\bar{j}}\rho^i\wedge\rho^j\eta^{\bar{i}}\theta_k g^{k\bar{j}}\Big) \tag{5.9}$$

with

$$V = g_{i\bar{j}}\Big(\rho^i_z\partial_{\bar{z}}\phi^{\bar{j}} + \rho^i_{\bar{z}}\partial_z\phi^{\bar{j}}\Big). \tag{5.10}$$

The second term is also topological since it is a differential form.

Let us try to explain the twisting resulting A model from another point of view. The $\mathcal{N}=2$ supersymmetry has the following charges

$$\{Q_+,\bar{Q}_+\} = H+P,\ \{Q_-,\bar{Q}_-\} = H-P, \tag{5.11}$$

and the chiral symmetries (or R-symmetry more properly.)

$$L:\psi^j_- \to e^{\mathrm{i}\alpha}\psi^j_-,\ \psi^{\bar{j}}_- \to e^{-\mathrm{i}\alpha}\psi^{\bar{j}}_- \tag{5.12}$$
$$R:\psi^j_+ \to e^{\mathrm{i}\alpha}\psi^j_+,\ \psi^{\bar{j}}_+ \to e^{-\mathrm{i}\alpha}\psi^{\bar{j}}_+, \tag{5.13}$$

corresponding to anomalous currents $J_\pm = g_{i\bar{j}}\psi^i_\pm\psi^{\bar{j}}_\pm$. Supercharges have definite helicity and have indices as spinor representations under Lorentz group. They transform as, with the group structure U(1)$_L\times$U(1)$_R\times$SO(2)

$$(1,0,1/2)\oplus(0,1,-1/2)\oplus(-1,0,1/2)\oplus(0,-1,-1/2). \tag{5.14}$$

The twisting procedure can regarded as a redefinition of Lorentz group generator, say T, as

$$T' = T + \frac{1}{2}U_L - \frac{1}{2}U_R. \tag{5.15}$$

Redefining the energy stress tensor essentially changes the representation of supercharges and the entire theory. As a consequence, we have 2 singlet supercharges Q_+ and $\bar{Q}_-$ and their sum $Q_+ + \bar{Q}_- = Q$ is nilpotent.
A third approach, maybe the most technically transparent one, is adding the following term to the Lagrangian

$$A_z\psi^i_-\psi_{-i} + A_{\bar{z}}\psi^i_+\psi_{+i}, \tag{5.16}$$

and set $A_z = -\mathrm{i}\omega_z/2$ and $A_{\bar{z}} = \mathrm{i}\omega_{\bar{z}}/2$. This extra term essentially redefines the energy stress tensor.

Despite several ways provided here to explain the twisting procedure, we think it is worth noting that in Witten's groundbreaking seminal works [6, 7], at first he merely constructed models based on the consideration that he needed a singlet fermion charge, that is to say, in principle we need not to thoroughly understand twisting, but merely consider models A and B variations of SUSY sigma model.

Besides, twisting is not a way to rewrite the same model. The original one, the A model and the B model are different models. The interesting part lies in A and B are mirror pairs and some correlation functions in the original model can be calculated using topological methods in A or B model.

There is maybe another way to think of these twisted models. In the Yang-Mills story, we start with the elegant Yang-Mills Lagrangian. After gauging fixing, we end up with a much more ugly action with a lot of field contents. Similar events happen in twisted models, where we have spin-0 and spin-1 fermions. Is it possible that those models are some gauge-fixed version of other elegant theories beyond?

To summarize, in this note we first introduce the topological field theories we are concerned about. The necessary ingredient, BRST symmetry is reviewed. We further examine what is the implication given a BRST charge. The following is an example that construct 2 topological field theories using twisting trick. Some different aspects and comments are supplemented to close the section.

References

[1] E. Witten, *Mirror Manifolds and Topological Field Theory*, arXiv: hep-th/9112056 (1991).

[2] D. Birmingham, M. Blau, M. Rakowski, and G. Thompson, *Topological Field Theory*, Phys. Rep. **209**, 129 (1991).

[3] S. Weinberg, *The Quantum Theory of Fields*, Volume II (2005).

[4] M. E. Peskin, and D. V. Schroeder, *An Introduction to Quantum Field Theory* (1995).

[5] S. Cordes, G. Moore, and S. Ramgoolam, *Lectures on 2D Yang-Mills Theory, Equivariant Cohomology and Topological Field Theories*, arXiv: hep-th/9411210 (1994).

[6] E. Witten, *Topological Quantum Field Theory*, Comm. Math. Phys. **117**, 353 (1988).

[7] E. Witten, *Topological Sigma Models*, Comm. Math. Phys. **118**, 411 (1988).