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# Spectral Flow & 1-loop amplitudes

Outline:
- Chiral Symmetry & conserved currents
- Spectral flow: Integral form of Anomaly
  - Laughlin's Argument & Edge anomaly

} 1+1 d U(1)

- Perturbative calculation
- Higher dimension, polygon diagrams & Anomaly Cancellation?

if have time.

## Chiral symmetry (establish notation)

- Free dirac particles in an external field

$$S = \int d^2x \, i\bar{\psi}\gamma^\mu D_\mu \psi \qquad D_\mu = \partial_\mu - iqA_\mu$$

using rep $\gamma^0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ $\gamma^1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ $\gamma = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- Point is, helicity eigenstates decouple

$$\mathcal{L} = \underbrace{i\psi_R^\dagger(D_0 + D_1)\psi_R}_{\text{ECM: these are right movers}} + \underbrace{i\psi_L^\dagger(D_0 - D_1)\psi_L}_{\text{left movers}} \qquad \psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}.$$

$\therefore$ particle no. seperately conserved

$$N_R = \int d^2x \, \psi_R^\dagger\psi_R \qquad N_L = \int d^2x \, \psi_L^\dagger\psi_L$$

(broken by mass term)

- Really a consequence of the symmetries

| | U(1) | Chiral |
|---|---|---|
| | $\psi \to e^{i\alpha}\psi$ | $\psi \to e^{i\beta\gamma^5}\psi$ |
| current | $j^\mu = \bar{\psi}\gamma^\mu\psi$ | $j^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$ |
| charge | $N_R + N_L$ | $N_R - N_L$ |

- In quantum theory, sum remains conserved, but the difference is not - sourced by electromagnetic field.

## Spectral Flow

- For first derivation, we introduce small background electric field

$$A_0 = 0 \qquad A_1 = \text{const in space, slowly varying in time}$$

- Electrons will be adiabatically transported as energy levels shift.
- Energy levels:

$$H = \int dx \left\{ -i\psi_R^\dagger D_1 \psi_R + i\psi_L^\dagger D_1 \psi_L \right\}$$

since $D_1 = \partial_1 - igA_1$ ($A_1$ const) $D_1$ eigenstates are $\partial_1$ eigenstates

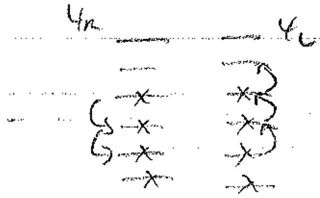
$$\psi_{R(L)} = e^{ikx} \qquad k = \frac{2\pi n}{L}$$

$$\varepsilon_{R(L)} = \pm(k - gA_1)$$

← compactified on spatial circle of length $L$ to discretize the spectrum.

- As $A$ increases $\Delta A_1 = \frac{2\pi}{gL}N$ spectrum sent to itself.

$$\varepsilon_R \to \varepsilon_R - \frac{2\pi N}{L} \qquad \varepsilon_L \to \varepsilon_L + \frac{2\pi N}{L}$$




- Rightmovers dissappear into dirac sea
- Leftmovers raise up out of it

$$\Rightarrow \Delta(N_R - N_L) = -2N$$

This is the instanton no. of the field configuration

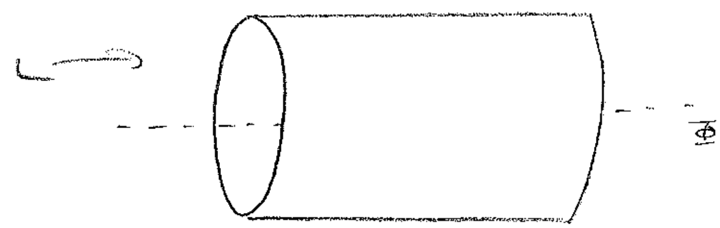
$$\frac{g}{\pi}\int d^2x\, \epsilon^{\mu\nu} F_{\mu\nu} = \frac{g}{\pi}\int d^2x\, \partial_0 A_1 = \frac{g}{\pi}L(-\Delta A_1) = -2N$$

$$\Rightarrow \qquad \Delta(N_R - N_L) = \frac{g}{\pi}\int d^2x\, \epsilon^{\mu\nu} F_{\mu\nu}$$

integrated form of anomalous non-conservation of chiral charge

## Laughlin's Argument

- Formally very similar to Laughlins derivation of QHE using flux insertion.
- Consider QH bar w/ $\perp$ magnetic field




periodic BC's in y-dir.
radial magnetic field.

- Idea is to insert a flux adiabatically through center & examine the spectral flow.

$$H = \frac{-1}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2m}\left(-i\frac{\partial}{\partial y} - Bx - A_y\right)^2 \qquad g = c = \hbar = 1$$

- Can solve eigenvalue problem by diagonalizing $\frac{\partial}{\partial y}$

$$\psi = e^{ik_y y}\,\psi(x) \qquad k_y = \frac{2\pi n}{L}$$

- $$H = \frac{-1}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2m}\left(k_y - A_y - Bx\right)^2$$

SHO centered at $x_0 = \frac{k_y - A_y}{B} = \frac{2\pi n}{BL} - \frac{A_y}{B}$ $\qquad \varepsilon = \hbar\omega_c(n+\tfrac{1}{2})$, $\omega_c = \frac{B}{m}$.

- Under $\Delta A_y = \frac{2\pi N}{L}$ energies fixed, but charge is transported.

$\Delta x_0 = -\frac{2\pi N}{BL}$ charges move N positions to left.

- charge removed from right edge, deposited on left.

- from point of view of edge theory, there is charge nonconservation.

$$\frac{1}{2\pi}\int_{\text{Edge}} d^2x\, \epsilon^{\mu\nu}F_{\mu\nu} = \frac{1}{2\pi}\int d^2x\, \partial_t A_y = \frac{1}{2\pi} L\Delta A_y = \Delta N.$$

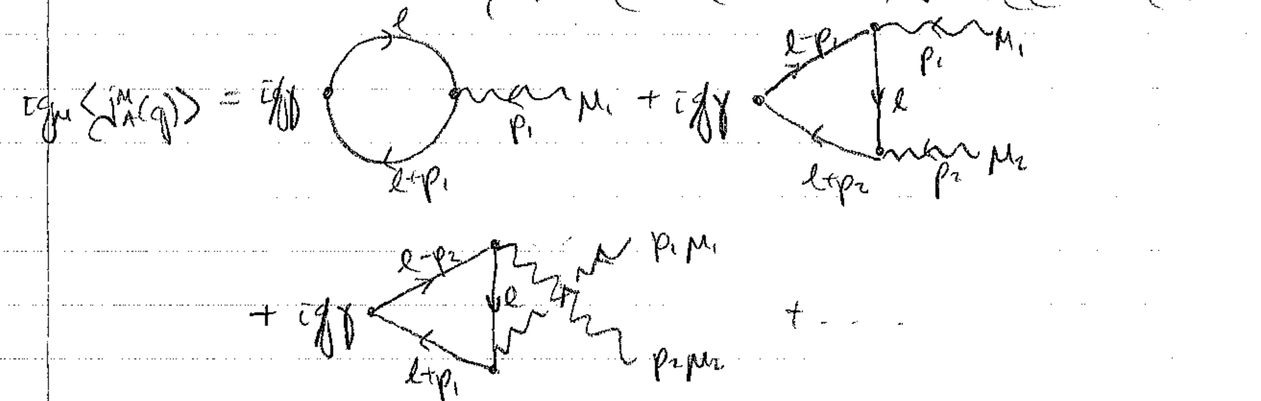
- from point of view of whole Bar charge is conserved
  - Anomaly inflow.

## Perturbative Calculation (1+1 d)

- really a local version

$$\partial_\mu \langle j_A^\mu \rangle = \frac{g}{\pi}\epsilon^{\mu\nu}F_{\mu\nu} \quad \text{would like to get at.}$$

- can find perturbatively in $g$ by evaluating polygon diagrams




$$\left( i q_\mu \langle j_A^\mu(q)\rangle = \frac{\int \mathcal{D}\bar\psi \mathcal{D}\psi \, i q_\mu j_A^\mu(q)\, e^{iS_0 + g\int \frac{d^2p}{(2\pi)^2} j^\mu(p)A(p)}}{\int \mathcal{D}\bar\psi\mathcal{D}\psi\, e^{iS_0+g\int\frac{d^2p}{(2\pi)^2} j^\mu(p)A(p)}} \right.$$

$$\left. = \sum_{n=0} \frac{g^n}{n!}\int \frac{d^2p_1\cdots d^2p_n}{(2\pi)^{2n}}\, i q_\mu \langle j_A^\mu(q) j^{\mu_1}(+p_1)\cdots j^{\mu_n}(+p_n)\rangle_c^0 A_{\mu_1}(p_1)\cdots A_{\mu_n}(p_n) \right)$$

- Treating $A_\mu$ as background field. Then integrate over $A_\mu$

  ☆Ask about this

- first diagram may be related to vacuum polarization diagram using $\gamma^\mu\gamma = -\epsilon^{\mu\nu}\gamma_\nu$

$$\langle j_A^\mu(q) j^\nu(-q)\rangle_c^0 = -\epsilon^\mu{}_\nu \Pi^{\nu\rho}(q)$$

$$i\Pi^{\mu\nu}(q) = (-1)(-ig)^2\int\frac{d^2\ell}{(2\pi)^2}\mathrm{Tr}\left[\gamma^\mu \frac{i}{\ell+q}\gamma^\nu\frac{i}{\ell}\right] \qquad \text{log divergent needs to be regularized.}$$

- For now lets be agnostic about choice of regulator

Index structure + dimension $\Rightarrow$ $i\pi^{\mu\nu} = i\left(A g^{\mu\nu} - B \frac{q^\mu q^\nu}{q^2}\right)$

for some $A$ & $B$ that will generally depend on regularization scheme.

(actually $B$ is regularization independent but we'll just leave it be).

- Lets see what this implies for conservation laws.

$$i q_\mu \langle j_A^\mu \rangle = i\,\epsilon^{\mu\nu} q_\mu \pi_{\nu\rho}(q) A^\rho(q)$$
$$= iA\,\epsilon^{\mu\nu} q_\mu A_\nu = A\,\epsilon^{\mu\nu} F_{\mu\nu}$$

$$i q_\mu \langle j^\mu \rangle = i\,\pi^{\mu\nu} q_\mu A_\nu = i(A-B)\, q^\mu A_\mu .$$

- So no gauge anomaly $\Rightarrow$ $A=B$
  no chiral anomaly $\Rightarrow$ $A=0$.

cannot do both at same time

Gauge inv. regulator $\Rightarrow$ Chiral anomaly

- for example w/ dim reg $A=B=\frac{e}{\pi}$ $\Rightarrow$ $\partial_\mu \langle j_A^\mu \rangle = \frac{e}{\pi}\epsilon^{\mu\nu} F_{\mu\nu}$ as before.

- we know higher order diagrams cannot contribute because are explicitly convergent

- also have attached diagramatic argument if people want to see it.

- Similar structure holds in higher dimensions
- Generally
  - Chiral anomaly in $d=2n$ dimensions
  - come from $(n+1)$-gon diagrams.

that is, $\partial_\mu \langle j^{\mu a}_A \rangle = -\frac{g^n}{16\pi^2} \epsilon^{\mu_1 \cdots \mu_d} \, \mathrm{Tr}\left(T^a F_{\mu_1\mu_2} \cdots F_{\mu_{d-1}\mu_d}\right)$.

## Anomaly Cancellation - The Standard Model.

- When have Chiral gauge fields (L + R handed particles in diff reps of gauge group)

  Chiral anomaly $\Rightarrow$ Gauge anomaly
  which must be zero for consistency.

  since $\sim \mathrm{Tr}\, T^a F \wedge F \sim \mathrm{Tr}\, T^a \{T^b, T^c\} F^b \wedge F^c$

  + R+L particles contribute w/ opposite sign to gauge anomaly

  $$\left( j^{\mu a} = \bar{\psi} \gamma^\mu T^a_L \frac{1+\gamma^5}{2} \psi + \bar{\psi} \gamma^\mu T^a_R \left(\frac{1-\gamma^5}{2}\right) \psi \right)$$

  we must have $\mathrm{Tr}\, \gamma^5 T^a \{T^b, T^c\} = 0$

- In standard model have
  $t^a$ ← SU(3) generators act same on R+L
  $\tau^a$ ← SU(2) generators act only on $\psi_L$
  $Y_{R+L}$ ← U(1) hypercharge.

- A lot of identities trivial
  - since $t^a$ not chiral need at least one factor of $Y$ or $\tau^a$
  - $\{t^a, t^b\} = \delta^{ab}$ $\quad \{\tau^a, \tau^b\} = \delta^{ab}$
  - $\mathrm{Tr}\, t^a = 0$ $\quad \mathrm{Tr}\, \tau^a = 0$

· Only nontrivial restrictions:

from:

$\mathrm{Tr}\,\gamma^5 \underset{\sim}{y}\{\underset{\sim}{y},\underset{\sim}{y}\}$ $\qquad \mathrm{Tr}\,\gamma^5 \underset{\sim}{y}^3 = 0 \quad \Rightarrow \quad \sum y_L^3 - \sum y_R^3 = 0$

$\mathrm{Tr}\,\gamma^5 \underset{\sim}{y}\{t^a,t^b\} = \delta^{ab}\,\mathrm{Tr}\,\gamma^5 \underset{\sim}{y}$ $\qquad \mathrm{Tr}\,\gamma^5 \underset{\sim}{y} = 0 \quad \Rightarrow \quad \sum y_L - \sum y_R = 0.$

$\mathrm{Tr}\,\gamma^5 \underset{\sim}{y}\{\tau^a,\tau^b\} = \delta^{ab}\,\mathrm{Tr}\,P_+ \underset{\sim}{y}$ $\qquad \mathrm{Tr}\,P_+ \underset{\sim}{y} = 0 \quad \Rightarrow \quad \sum y_L = 0.$