Chiral Phase transition

$SU(N_c)$ gauge theory w/ N_f quarks

$$\mathcal{L} \supset \sum_{i=1}^{N_f} \bar{\psi}^i (i\cancel{D}) \psi_i$$

$$= \sum_{i=1}^{N_f} \left\{ \bar{\psi}^i_L (i\cancel{D}) \psi_{L,i} + \bar{\psi}^i_R (i\cancel{D}) \psi_{R,i} \right\}$$

Global Symmetry:

$$G_f = U(N_f)_L \times U(N_f)_R$$

$$= U(1)_V \times U(1)_A \times SU(N_f)_L \times SU(N_f)_R$$

Topic: T-dependence of manifest G_f symmetry?

Comments:

- $U(1)_A$ is anomalous

$$\partial_\mu J^{\mu 5} \sim g^2 N_f \, \mathrm{tr}\, \tilde{F} F$$

$SU(N_c)$ gauge fields

But anomaly still respects $\mathbb{Z}_{N_f} \subset U(1)_A$

Why? $\psi \to e^{i\alpha\gamma_5/2}\psi$ under $U(1)_A$

In instanton bkgd,

$$\underbrace{\langle \psi_R \cdots \psi_L \rangle}_{2N_f} \neq 0$$

$$\Big\downarrow U(1)_A$$

$$\underbrace{e^{i\alpha N_f}}_{=1 \text{ if } \alpha = \frac{2\pi M}{N_f}} \langle \cdots \rangle$$

- Anomaly vanishes in planar limit:

$$N_c \to \infty, \quad \lambda = g^2 N_c \text{ fixed}$$

Because quark loops give subleading rel. to glue:

$$\partial_\mu J^{\mu 5} \sim \frac{\lambda}{N_c} \to 0$$

So, in presence of anomaly:

$$G_F' = U(1)_V \times \mathbb{Z}_{N_f} \times SU(N_f)_L \times SU(N_f)_R$$

Aside: spontaneous breaking of Z_{N_F} gives no NG bosons, but in $N_c \to \infty$ limit, there is one, η'. Instantons give it a mass

Does T-dependent vacuum respect G_F'?

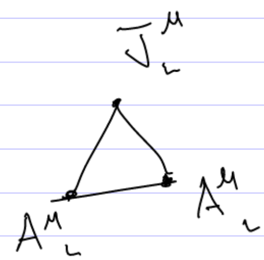
Already answered @ $T=0$:

Strong dynamics leads to spontaneously broken chiral symm. in IR

t'Hooft anomaly matching:

G_F' has t'Hooft anomalies:

$$\left(SU(N_F)_{L,R} \right)^3$$



Conserved along RG flow. But, in IR we have confinement. "Can check" that confined spectrum can't reproduce anomaly unless G_F' is spontaneously broken

What about $T > 0$?

Lattice shows a $T_{ch} < \infty$

$\uparrow$

independent of g^2

at which G_F' is restored.

Assume so, try to learn if 1st or 2nd order

Effective theory of order parameter:

$$\phi^i{}_j \sim \langle \bar{\psi}_L^i \psi_{R,j} \rangle$$

Transforms under $U(1)_A \times SU(N_f)_L \times SU(N_f)_R$

as

$$\phi \to e^{i\alpha} U_L^+ \phi U_R$$

Most general, renormalizable $\mathcal{L}(\phi)$:

$$\mathcal{L}_\phi = \frac{1}{2} \mathrm{tr}(\partial_\mu \phi^+ \partial^\mu \phi) - \frac{1}{2} M_\phi^2 \mathrm{tr}(\phi^+ \phi)$$

$$- \frac{\pi^2}{3} g_1 (\mathrm{tr}\, \phi^+ \phi)^2 - \frac{\pi^2}{3} g_2 \mathrm{tr}(\phi^+ \phi)^2$$

$g_2 \geq 0$, $g_1 + g_2/N_f \geq 0$ for stability

Does not reproduce anomaly:

$$\mathcal{L}'_\phi = c\left(\det\phi + \det\phi^\dagger\right)$$

$$\sim \langle \underbrace{\bar{\psi}_L \psi_R \cdots \bar{\psi}_L \psi_R}_{2N_f} \rangle$$

Finite T, compactify Euclidean time,
integrate out all massive (EMT) modes
and focus on 3D theory

Since phase transition occurs @ $T = T_{ch}$

$$M_\phi^2(T) \sim M_\phi^2 (T - T_{ch})$$

$T < T_{ch}$, $M_\phi^2(T) < 0$

minimum is at $\phi \neq 0$

SSB

$T > T_{ch}$ $M_\phi^2(T) > 0$, $\langle \phi \rangle = 0$

@ $T = T_{cr}$, $m_\phi^2 = 0$

"Widely believed that" if an IR stable fixed point exists for $\mathcal{L}_\phi(T = T_{cr})$ then transition is 2^{nd} order. Else, 1^{st} order

Why?

Speculation: · 2^{nd} order described by CFT, i.e. fixed point?
· 2^{nd} order exhibits universality?
Stable fixed point means independent of UV

Anyway: calculate β-functions of g_1, g_2 in $4-\epsilon$ dimensions:

For $C = 0$

$$\beta_1 = -\epsilon g_1 + \frac{(N_f^2+4)}{3} g_1^2 + \frac{4N_f}{3} g_1 g_2 + g_2^2$$

$$\beta_2 = -\epsilon g_2 + 2 g_1 g_2 + \frac{2N_f}{3} g_2^2$$

No stable fp if $N_f \geq \sqrt{3}$
Stable if $0 < N_f \leq \sqrt{2}$

$c \neq 0$

W/o details, for

$N_f \geq 3$ c doesn't affect 1st order

$N_F = 2$, c can push transition to 2nd order

What is c?

c couples to anomaly of $U(1)_A$ ~ instantons

Assume $c(T) \sim d_I(T)$

T-dependent instanton density

$c(0) \sim \mathcal{O}(1)$ since $c(0)$ gives mass to η', separating it from other NGs.

$c(T \to \infty) \to 0$ since instantons of size $\rho > \frac{1}{T}$ don't fit into compact time

So, at high T, approx. restoration of $U(1)_A$.

Since anomaly gives η' mass apart from other NGs, this has affects on spectrum