Detecting Fractional Charge and Statistics in the FQHE

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June 12, 2014

1 Introduction

There are three quantities one would hope to measure to validate the quasiparticle picture of the FQHE, as well as the chiral Luttinger liquid model of the edge. These are the *fractional charge*, the *fractional statistics*, and the *anomalous scaling exponents* associated with tunneling of edge currents through a point contact. In this paper I will discuss the detection of fractional charge via shot-noise measurements as well as the theory of the Quantum Hall Interferometer and its applications to measuring fractional charge and statistics along with associated experiments.

2 Shot Noise and Fractional Charge

The first convincing experiments to detect fractional charge came from measuring shot noise in tunneling current at a point contact.

2.1 Experimental method

A 2DEG is created with a GaAs/AlGaAs heterojunction cooled down to 50 mK. The Hall bar is equipped with a back gate and sides gates used to constrict the quantum fluid so that the two edges are brought close together, this is called a quantum point contact (QPC). The gates can be used to vary the transmission of the point contact. If there is a potential difference between the two edges there will be a tunneling current. This current can be measured and fed through an amplifier and then into a spectrum analyzer which measures current fluctuations.

There are two fundamental sources of noise, *shot noise* and *thermal noise*. There are also lots of other non-fundamental sources of noise, such as instrument noisee. The noise

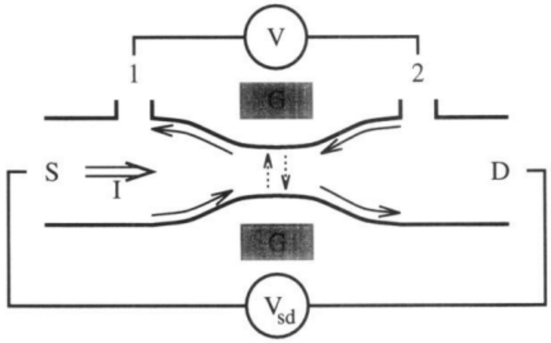

FIG. 1. Schematic of quantum Hall bar with gated constriction (G) separating source from drain. Of interest are nonequilibrium fluctuations in the current, I, and voltage, V, in the presence of an applied source-to-drain voltage, V_{sd}.

due to the amplifier can be measured independently and subtracted from the total noise in order to obtain the shot noise associated with the tunneling quasiparticles.

2.2 Background and Results

Shot noise is due to the discrete nature of your signal carriers. For example, this could be due to the discrete energy carried by a photon or, in our case, the discrete nature of charge carriers in a current. If you assume quasiparticle tunneling to be a Poisson process, and do not take into account possible nontrivial statistics, then at finite temperature, the spectral density function is simply:

$$S(\omega) = 2QI_B \tag{1}$$

The *spectral density function* is a function, which integrated over a range of frequencies, gives you the contribution to the variance of your signal from that range of frequencies. At non-zero temperatures there is also Nyquist-Johnson noise due to thermal fluctuations. Furthermore, due to Coulomb interactions between charges, their motion is not independent and thus this is not truly a Poisson process. A more general expression for the *zero frequency* spectral density of the *total noise* in a single channel is

$$S = 2g_0t(1-t)\left(QV\coth\left(\frac{QV}{2k_BT}\right) - 2k_BT\right) + 4k_BTg_0t \tag{2}$$

which smoothly interpolates between the thermal noise at equilibrium (V=0) and a linear current dependence when the voltage is much larger than k_BT/Q, for which $S \approx 2IQ(1-t)$.

The constant t is the transmission of the QPC, which is the ratio between its conductance, G, and the quantum conductance $g_0 = e^2/h$. The noise as a function of backscattered current is plotted below together with a fit from Eq. 2 using $Q = e/3$ for a Hall bar in the

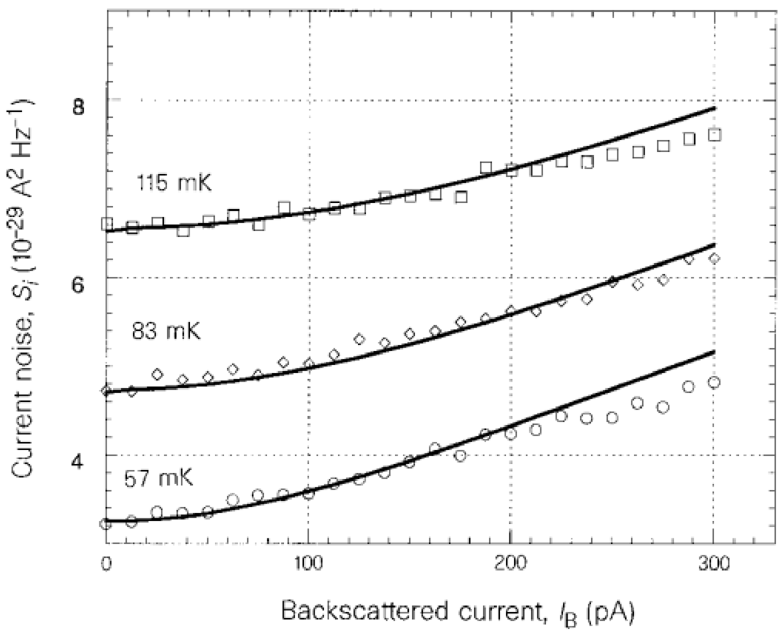

Figure 4 Quantum shot noise as a function of backscattered current, I_B, in the FQH regime at $\nu = \frac{1}{3}$, for three different temperatures and a constant transmission coefficient, $t = 0.8$, through the QPC.

$\nu = 1/3$ regime. This data was obtained from reference [4] in an experiment conducted in 1997. Similar results were found in a similar experiment also published in 1997.

2.3 Controversy

Despite the apparent simplicity of these experiments, they are not without controversy, especially when extended into other regimes of the experimental parameters. More recent measurements from the same group which were published in 2010 [6], suggest that this picture is not so simple. This work was "motivated by an attempt to improve the accuracy of [their] previous measurements and tighten the data points with more sensitive measurements, thus allowing [them] to determine the charge in a previously inaccessible low energy and very weak backscattering regime". In this regime, "tunneling charges across a constriction were found to be significantly higher than the theoretically predicted fundamental quasiparticle charges". (It is interesting to note that in 2008 this same group published a paper in nature claiming that they had observed charge e/4 tunneling via shot noise experiments in the $\nu = 5/2$ state which has been cited about 230 times, meanwhile, their paper in 2010 which show much more interesting and somewhat contradictory behavior has only been cited 23 times.) A summary of their results for the $\nu = 5/2$ state is depicted in the figure below. Similar dependence of the charge on the transmission was found in other filling fractions.

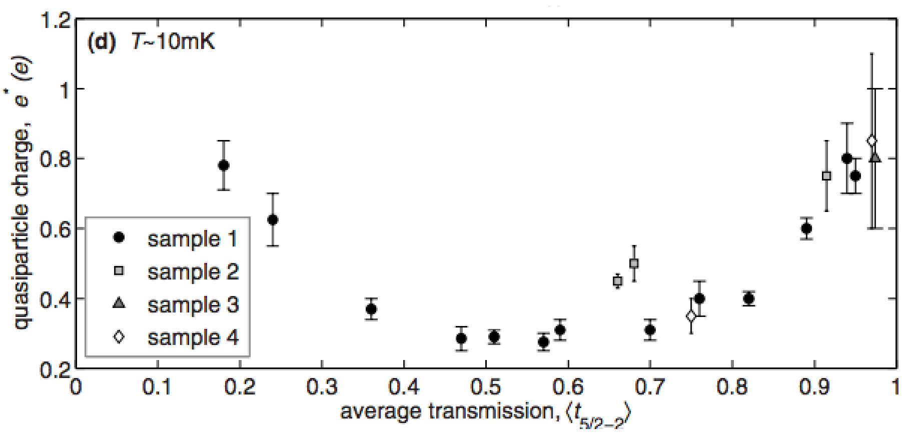

Figure 1: Evolution of the tunneling charge in the $\nu = 5/2$ state as the transmission is varied

In addition to transmission dependence of the quasiparticle tunneling charge, they also found dependence on temperature. At low temperatures they found that the tunneling charge is also larger the fundamental quasiparticle charge. These results, for the $\nu = 5/2$ state, are depicted in the figure below

The enhancement of charge can be attributed to backscattering of a mixture of integer multiples of the fundamental charge, but is not fully understood and contradicts the chiral Luttinger Liquid predictions for tunneling. These results not only have interesting theoretical implications, but are also very relevant for future experiments, such as interference experiments in which the charge of the weakly scattered quasiparticle is usually taken for granted. A better understanding of the tunneling processes that take place between quantum Hall edges in the quantum point contact is needed to interpret the shot-noise results.

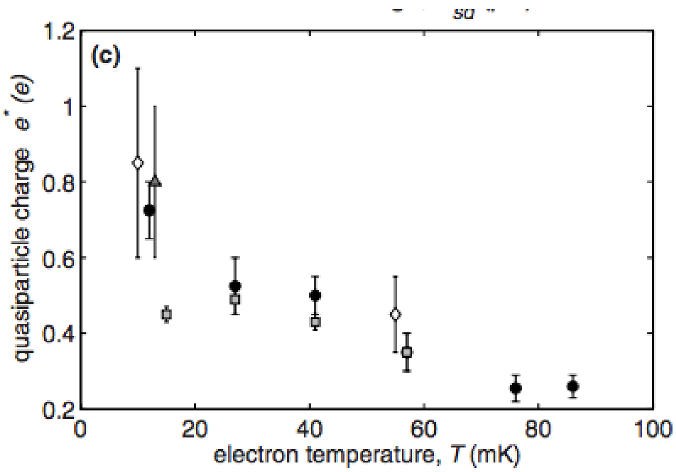

Figure 2: Evolution of the tunneling charge in the $\nu = 5/2$ as the temperature is varied

NOTE: Local measurements of quasiparticle charge in the bulk of a $\nu = 5/2$ state have found more universal results for quasiparticle charge [7].

3 Quantum Hall Interferometer

3.1 Qualitative Aspects

The two point-contact quantum Hall interferometer is a device that, at least theoretically, should allow direct observation of fractional charge *and* statistics, as well as provide tests for the chiral Luttinger liquid model of the edge. One of the earliest proposals for such a device came in 1997 from Chamon, Freed, Kivelson, Sondhi and Wen [1]. The device consists of three principal components, a narrow quantum Hall bar with tunable point contacts, a back gate which allows the electron density in the Hall bar to be varied uniformly and a central gate which can deplete the charge density in the central region by application of a voltage.

In this setup, a voltage difference is applied between the two edges and the resulting backscattered current is measured. The backscattered current can take two paths; path two leads the current around a section of the bulk quantum hall fluid and path one does not. The two currents interfere and so the total current will oscillate with the phase difference between the paths. The phase oscillations can be broken into three parts, Aharanov-Bohm oscillations due to the magnetic flux piercing the Hall bar, statistics oscillations due to

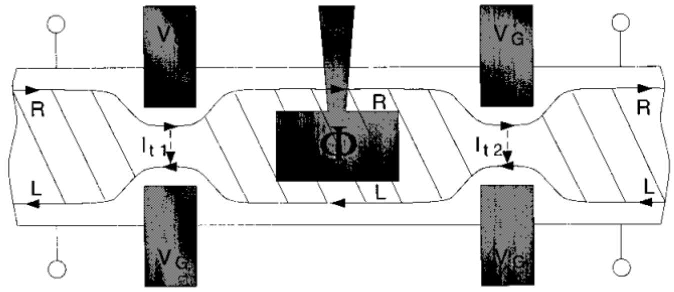

Figure 3: Two-point contact quantum Hall interferometer

the phase acquired when the quasiparticles traveling along path two braid around the quasiparticles in the bulk and finally, so-called "Fabry-Perot" oscillations due to varying the source-drain voltage (which we will not discuss in this paper).

The Aharanov-Bohm phase associated with a charge q circling a magnetic flux, BA, where B is the magnetic field and A is the area is

$$\theta = \frac{2\pi q B A}{hc} \tag{3}$$

if we define $\Phi_0 = hc/e$ and $\Phi^* = (e/q)\Phi_0$ then naively we would expect the quasiparticle current to undergo oscillations with period

$$\Delta B^* = \frac{\Phi^*}{A} \tag{4}$$

However, this conclusion is not necessarily correct. This is because the quasiparticles are really excitations of the bulk electron condensate which is the relevant *vacuum*. If this vacuum is not invariant, then we can not expect to be able to make arguments based solely on the AB phase of the quasiparticles. We must distinguish between two different cases

3.1.1 Field sweeps at fixed particle number

In this case, we can imagine two different possible outcomes from changing the magnetic field, depending on the microscopic properties of the Hall bar. The first possibility is that as the B field is increased the Hall droplet shrinks uniformly because $l_B = \sqrt{\hbar c/eB}$. In this case the flux through the whole droplet would be unchanged and you would not observe oscillations with period ΔB^*. Another possibility is that the droplet does not shrink. In this case, quasi holes would be created in the bulk, one for each additional flux quantum, and so there would now be a statistical phase $\theta^* = -2\pi/m$ associated with braiding around the quasi hole (if you are in a $\nu = 1/m$ state) and so once again you would not see oscillations with period ΔB^*.

3.1.2 Field sweeps at fixed filling factor

Recall that the spatial density of states in the bulk is

$$\rho_{states} = \frac{1}{2\pi l_B^2} = \frac{eB}{2\pi\hbar c} \tag{5}$$

and that the filling factor is given by

$$\nu = \frac{\rho_e}{\rho_{states}} \tag{6}$$

because the Hall bar is equipped with an overall back gate, the electron density in the bulk can be varied. If, while we sweep the magnetic field, the back gate voltage is adjusted so as to keep the filling factor constant, then the bulk will be invariant and we can concentrate on the properties of the quasiparticles. In this case the resulting periodicity would be ΔB^* and an observation of this period would constitute an observation of fractional charge.

If there are N quasiholes present, the interference phase would be modified to be

$$2\pi(BA/\Phi^* - N/m) \tag{7}$$

To detect fractional statistics, one could, in principle, adjust the voltage of the central gate to deplete the charge density in the center in steps of $1/m$. One would then expect to see conductance oscillations with a period of m steps.

4 Tunneling at Point Contacts for $\nu = 1$

In this section we will study the tunneling of edge currents in the $\nu = 1$ state by a single point contact. We will then show how this analysis can be extended to the two point contact case and sketch the derivation of the tunneling current in the weak backscattering regime. The discussion will follow reference [1], section IV.

4.1 Single point contact

In the $\nu = 1$ case the edge can be described by a model of free chiral fermions scattering from a point contact. The Hamiltonian for this system is

$$H = \int dx \left[\psi_R^\dagger(x) \left(-i\frac{\partial}{\partial x} - \frac{\omega_0}{2} \right) \psi_R(x) + \psi_L^\dagger(x) \left(i\frac{\partial}{\partial x} + \frac{\omega_0}{2} \right) \psi_L(x) + 2\pi\delta(x) \left(\Gamma_i \psi_L^\dagger \psi_R(x) + H.C. \right) \right] \tag{8}$$

One can then solve for the scattering matrix which relates the incoming modes ψ_{R-} and ψ_{L+} to the outgoing modes ψ_{R+} and ψ_{L-}. It is given by

$$S = \begin{pmatrix} t_i & r_i \\ -r_i^* & t_i \end{pmatrix} \tag{9}$$

where the transmission and reflection amplitudes are given by

$$t_i = \frac{1 - \pi^2 |\Gamma_i|^2}{1 + \pi^2 |\Gamma_i|^2} \qquad \text{and} \qquad r_i = \frac{-i2\pi\Gamma_i^*}{1 + \pi^2 |\Gamma_i|^2} \tag{10}$$

4.2 Two point contacts

When there are more than one point contacts, it is more convenient to use the transfer matrix which relates the modes on the left side of the barrier ψ_{R-} and ψ_{L-} to the modes on the right side of the barrier ψ_{R+} and ψ_{L+}, which we will label by M_i. In addition, we also have to take into account the extra accumulated phase by the modes between the point contacts. If the distance between the point contacts is a, then we will have to multiply by the matrix

$$D = \begin{pmatrix} e^{ia\omega} & 0 \\ 0 & e^{-ia\omega} \end{pmatrix} \tag{11}$$

Now the modes to the right of both point contacts is related to the modes to the left of both point contacts by

$$\begin{pmatrix} \psi_{R+} \\ \psi_{L+} \end{pmatrix} = M_2 D M_1 \begin{pmatrix} \psi_{R-} \\ \psi_{L-} \end{pmatrix} \tag{12}$$

By expanding the matrices, one can obtain the transmission amplitude

$$t(\omega) = \frac{t_1 t_2}{1 + r_1 r_2^* e^{2i\omega a}} \tag{13}$$

and the associated transmission coefficient

$$T(\omega) = \frac{|t_1|^2|t_2|^2}{1 + |r_1|^2|r_2|^2 + (r_1 r_2^* e^{2i\omega a} + r_1^* r_2 e^{-2i\omega a})} \tag{14}$$

If there is a potential difference between the edges with an associated frequency $\omega_J = qV/\hbar$ then the Fermi-Dirac distribution will give the relative occupation of the two modes

$$n^{R,L}(\omega) = \frac{1}{e^{\beta(\omega \mp \omega_J/2)} + 1} \tag{15}$$

In this simple model the total current flowing through the droplet is the total right moving current minus the total left moving current

$$I = e \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} T(\omega) \left[n^R(\omega) - n^L(\omega) \right] \tag{16}$$

We are interested in finding the tunneling current in the zero-temperature weak backscattering limit. In the limit of no backscattering ($r_1 = r_2 = 0$) the above integral reduces to $I_{Total} = (e^2/h)V$. The backscattering current is then given by $I_B = I_{Total} - I$ where I is the current obtained from equation 16 in the limit $|r_1|, |r_2| \ll 1$. The result is

$$I_B = q|\Gamma_{\text{eff}}|^2 2\pi\omega_J \tag{17}$$

where

$$|\Gamma_{\text{eff}}|^2 = |\Gamma_1|^2 + |\Gamma_2|^2 + (\Gamma_1\Gamma_2^* + \Gamma_1^*\Gamma_2)\frac{\sin(\omega_J a)}{\omega_J a} \tag{18}$$

Here the Aharanov-Bohm phase has been incorporated into the scattering amplitude

$$\Gamma_2 = \bar{\Gamma}_2 e^{-i2\pi qBA/hc} \tag{19}$$

Thus we see quite clearly that the backscattered current will oscillated with period ΔB^*. The extension to fractional quantum hall states can be done perturbatively, and gives a similar result. The AB phase is the same, but the dependence on voltage is different and depends on the Luttinger liquid exponent, which for Laughlin states, is simply $g = 1/m$. The inclusion of finite temperature effects is discussed in section V of [1].

5 Interferometry Experiments

A typical quantum Hall interferometer, for example the one in [9], consists of a 2D electron gas trapped in an AlGaAs/GaAs heterojunction. Au/Ti gates are deposited on the surface and the whole thing is mounted on a sapphire substrate and cooled down to 10.2 mK. The sample has front gates which create the point contacts and an overall back gate, but no central gate as discussed in [1]. The area surrounded by the interference paths of the edge modes is approximately $2.14\mu m^2$ for the $\nu = 1/3$ regime. The electron density in the bulk is $n_B = 1.25 \times 10^{11} cm^{-2}$, which implies approximately 3500 electrons in the central island. Then experimental parameters like the external magnetic field and the back gate voltage are varied and oscillations in the conductance are measured.

5.1 Results and interpretation

I invite the interested reader to examine the papers [8] and [9] as well as others which are discussed in [2]. The results are difficult to interpret and it is not clear whether or not the observed oscillations are actually due to quasiparticle statistics, or due to other mechanisms [10]. Instead of trying to interpret these experiments I will just list some of the main experimental difficulties/complications that arise in these interferometry experiments. This is still a very active field of research both theoretically and experimentally, and we can expect that future research will be done to try to improve and understand these difficult experiments.

5.2 Experimental Complications

1. Tunneling, even with a single point contact, is not fully understood (as indicated by [6]).
2. Experimentally there is not such a sharp distinction between the quasiparticles which are localized in the bulk and the quasiparticles traveling around the edge.
3. The number of quasiparticles in the interferometer generally varies during the timescale of the experiment so the number of quasiparticles has to be averaged over when considering Aharanov-Bohm oscillations.
4. There is potentially complicated dependence of the area of the droplet on the magnetic field.
5. "*Interferometer experiments have led to puzzling results even in the integer regime, which have posed a challenge to our theoretical understanding*"[2].
6. For tunnel junctions with a weak bias voltage, Coulomb Blockade effects, as a function of both magnetic field and back gate voltage, are important, especially for small interferometer sizes similar to what were used in early experiments. In fact, in recent experiments,

Coulomb Blockade effects were identified as the dominant form of resistance oscillations for a 2 μm^2 device [10]. These effects seem less important for much larger devices of order 20 μm^2, but such large devices sizes are only just beginning to be explored. For the larger device sizes, oscillations consistent with AB predictions have been observed in the IQH regime where $\Delta B = hc/e$ for each integer filling fraction. In fact, at the end of their paper [10], they discredit the results obtained in [8] and [9].

7. "*There are clear signatures of non-universal effects*" – Woowon Kang

8. A paper published in 2009 [10], which studied interference effects in the integer quantum hall regime stated "*The questions of whether it is even possible to observe resistance oscillations that arise from pure AB interference in FPIs, and if so, in what regime, and how to distinguish the two mechanisms, have yet to be answered to our knowledge.*"

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