Color Superconductivity

Overview

- Color superconductivity is predicted phase of QCD at ultra high density & low temperature
  - in particular for this talk will take $T=0$ & $\mu=m$ 
  - treat quarks as massless to excellent approximation
  - Symmetry $SU(3)_c \times SU(3)_L \times SU(2)_L \times U(1)_B$

- Idea: along lines of normal superconductivity
  - attractive interactions among quarks
  - $\Rightarrow$ formation of diquark bound states
    - condensate

- Vev breaks symmetry
- Argue to diagonal subgroup
  - $SU(3)_c$ - simultaneous global flavor & color transformations.

- Called "color-flavor locked" or CFL phase

Outline: 

- Argue for CFL state
  - Consequences
  - Quantitative results for gap formation
    - Not really reliable since sensitive to choice of form factor, etc...

Mainly follows A. Rajaraman & Wilczek (1999)
Color-Flavor-Locked State

- really educated guesswork as to what's energetically favorable
  
  - we know in this regime, quarks exert weak attraction
    
    \[ \langle g_{i a} g_{j b} \rangle \langle \bar{g}_{i a} \bar{g}_{j b} \rangle \]
    
    \( i = \text{flavor} \)
    
    \( a = \text{color} \)
    
    \( \theta = \text{spin} \)
    
    Condensate should be \( \langle g_{i a} g_{j b} \rangle \langle \bar{g}_{i a} \bar{g}_{j b} \rangle \)
    
  - Argue spin 1 component will not compete w/ spin 0 component
    
    (consistency = 2 entire fermi surface cannot contribute sufficiently)
    
- parity preserving
  
  - simply don't see any sign of \( \gamma \), even when include
    
    interaction effects (see 1968 paper by e.g. Audis)
    
    So pick parity invariant state
    
    \[ \langle g_{i a} g_{j b} \rangle = \langle \bar{g}_{i a} \bar{g}_{j b} \rangle \]
    
- color antisymmetric channel is the attractive channel
  
  - P flavor antisymmetric as well
    
    So expect something like
    
    \[ \langle g_{i a} g_{j b} \rangle = \text{det} \cdot e_{i j} \]
    
  - turns out in correct solve self-consistent gap eqn & so need
    
    more general ansatz
    
    \[ \langle g_{i a} g_{j b} \rangle = \mu \cdot e_{i j} + \nu \cdot \bar{e}_{i j} \]
    
- State not invariant under individual color or chiral flavor rotation
  
  or \( U(1)_a \), but is under the diagonal subgroup
  
  \( \otimes \) looks \( \text{SU}(3)_c \times \text{SU}(3)_c \) & run \( \otimes \) \( \text{SU}(3)_c \) to \( \text{SU}(2)_c \)
    
  - hence called color-flavor locked phase.
Immediate Consequences - Single particle excitations

- will see that $B_u$ & $u$ gaps for the 9 types of quasiparticles
  - uniform over fermi surface so no low energy single particle excitations.
- Higgs mechanism.
  - Since color locked to flavor, no local symmetries remain
    => all gluons acquire mass.
- 8 boleb chiral generators, 1 broken U(1) generator.
  - $SU(3)$ octet - & singlet of Nambu-Goldstone bosons
- $SU(4)$ broken by instantons further broken to $Z_2$ by the condensate.
- Quark masses would break flavor symmetry
  - give the $SU(3)$ octet small masses + diminish symmetry of condensate.
  - $U(1)$ is still masses.

Determining the Gap

- Single gluon exchange $\rightarrow$ attractive interaction.
  - use model hamiltonian based on this color structure

$$H_z = 2K \sum (\delta_{[SS]} - \frac{1}{3} \delta_{[\pi\pi]} ) \delta_{[SS]} \delta_{[\pi\pi]}$$

$F$ symbolizes momentum dependent form factor for each leg of interaction to mock up asymptotic freedom.

$$F(p) = \left(1 + \exp\left(\frac{p^2 - \mu^2}{\Lambda^2}\right)\right)^{-1}$$

- will just kind of pick $A \mu \nu$, pretty arbitrary, so the calculation cannot be truly trusted at a quark-level, but
will hopefully give insight into what's happening.

- Now as in usual $CS$ theory, make mean field approx

\[
\langle \delta \phi \rangle = \frac{3}{2} \left( \phi - \phi_0 \right) + \phi_0 \delta \phi \quad \text{in small}\]

We parameterize the field by

\[
\phi_{\alpha \beta} = \frac{1}{2} (\delta_{\alpha \beta} + 3 \Delta_\alpha \Phi) \delta_{\alpha \beta} + \frac{1}{2} Q_{\alpha \beta} \delta_{\alpha \beta}
\]

since $\Delta_\alpha \Phi$ will turn out to be gaps.

- After this approx the total hamiltonian becomes

\[
H = \int d^4 x \left[ \left( \partial \Phi \right)^2 - m^2 \Phi^2 \right] + \frac{1}{2} \int d^4 x \left[ Q_{\alpha \beta} Q_{\alpha \beta} \right]
\]

where $Q_{\alpha \beta} = \Delta_\alpha \Phi \delta_{\alpha \beta} + \frac{1}{2} (\Delta_\alpha - \Delta_\beta) \delta_{\alpha \beta}$

Quadratic form in (e), so can diagonalize to find quark particle spectrum.

- First decompose into

\[
\phi_{\alpha \beta} = \frac{1}{\sqrt{N}} \sum_{\xi} \left[ \begin{array}{c} - \sin \theta \xi e^{i \varphi} \\ - \cos \theta \xi e^{i \varphi} \end{array} \right] \left( \left( \begin{array}{c} 1 \\ -1 \end{array} \right) \right)
\]

$\xi = 1, \cdots, 9$ is color-flavor index.

Diagonalizing $Q$ in color-flavor space, we find

\[
H = \sum_{\alpha \beta} (1 - \mu) \alpha_{\alpha \beta} Q_{\alpha \beta} + \sum_{\alpha \beta} (\mu - 1) \alpha_{\alpha \beta} Q_{\alpha \beta} + \sum_{\alpha} (1 - \mu) \beta_{\alpha} \bar{\beta}_{\alpha}
\]

\[
+ \frac{1}{2} \sum_{\alpha \beta} F(\alpha_{\alpha \beta} Q_{\alpha \beta} - \beta_{\alpha} \bar{\beta}_{\beta}) + \text{c.c.}
\]

where $\alpha_1 = \Delta, \alpha_2, \cdots, \alpha_n = \Delta$ are eigenvalues of $Q$.

Can be diagonalized through some unenlightening algebra, to

\[
H = \sum_{\alpha} \left( \sqrt{1 - \mu^2} + F(\Delta) Q_{\alpha} \right) \gamma_{\alpha} \gamma_{\alpha} + \left( \sqrt{\mu^2} + F(\Delta) Q_{\alpha} \right) \gamma_{\alpha} \gamma_{\alpha} + \mu \gamma_{\alpha} \gamma_{\alpha}.
\]

So we see physical gaps of $F(\Delta, 1) + F(\Delta, 0)$.
As in BCS theory, \( \Delta_1, \Delta_2 \) must satisfy a self-consistency condition that can be used to solve for them.

\[
\langle \Phi^i \Phi^j \rangle = \frac{2}{\sqrt{16}} \rho_{ij}
\]

Plug in \( \Phi \) in terms of \( \chi, \Phi^{\dagger} \) to evaluate in \( T=0 \) state,

\[
s = \frac{\imath \chi_1 \chi_2}{\sqrt{16}} = \frac{\imath \chi_1 \chi_2}{2} = 0.
\]

\[
\Rightarrow \Delta_1 + \frac{1}{2} \Delta_2 = \frac{\imath}{3} \mu G(\lambda_1), \quad \frac{1}{2} \Delta_1 = \frac{\imath}{3} \mu G(\lambda_2)
\]

\[
G(\lambda) = -\frac{1}{2} \frac{\lambda}{1 + \Phi(\lambda^2 + \Phi(\lambda^4 + \Phi(\lambda^8)))} \quad \text{(the value)}
\]

Numerical sol: Set \( \Lambda = \frac{1}{2} \), value at Chiral Gap is \( 0.4 \) GeV (arbitrary).

\[\Lambda = 0.8 \text{ GeV}, \quad \mu = 0.85 \text{ GeV.}\]

We are overestimating.

Only use 1 interaction, no take half.