Anomalies - KCTP Journal Club

QFT 1

\[ \chi = V - E + F \]

QFT 2

A crude computation allows you to distinguish 1 from 2 due to topology genus = 0 or 1

Anomalies - a useful tool for understanding some properties of a QFT which necessarily being able to compute everything perfectly.

General meaning: A symmetry is anomalous if it is a symmetry of the classical theory but is violated by quantum effects.

Why this happens:

\[ \langle \phi(x) \rangle \propto \int \mathcal{D} \phi \phi(x) e^{i S[\phi]} \]

measure may not be invariant under a transformation.
Then as many kinds of anomalies as there are types of symmetries.

Chiral anomaly
conformal anomaly
gauge anomaly
diffeomorphism anomaly

For there is a long list of applications of anomalies. Let me mention some of most important, weighty towards particle theory.

First, most basic example:

\[ \text{QCD} \quad 0 \]

quark kinetic terms:
\[ i \bar{\psi} D \psi = -i \bar{\psi} \gamma^\mu \partial_\mu \psi + i \bar{\psi} \gamma^5 \partial_5 \psi \]

mass terms:
\[ m \bar{\psi} \psi = m \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \psi - \bar{\psi} \gamma^\mu \gamma_5 \partial_\mu \psi \]

When \( m = 0 \) have
\[ \psi \to e^{i \alpha} \psi \quad \text{un} \]
\[ \bar{\psi} \to e^{i \beta} \bar{\psi} \quad \text{un} \]

on time like com, \( V(u) \)
\[ \text{Lagrangian:} \quad \frac{\partial}{\partial \phi} \int \partial_\mu \phi - \epsilon - \frac{\partial_\eta \phi}{\sqrt{2}} \]

Classically \( \partial_\mu \phi = 0 \) when \( m = 0 \)

1-loop effects:
\[ \Delta \phi = -\frac{\rho}{4 \pi^2} \int \frac{d^4 k}{k^2} \]
From $j^A_{\mu}$ comes $A^\mu_{\nu}$

The diagram is divergent and can't be regularized in a way that preserves gauge invariance and conservation of $j^A_{\mu}$.

Sounds like UV effect.

But opposite fields that can have a mass term consistent with $SU(2)$ symmetry can now UV are regularized by Pauli–Villars

...properties determined by $\partial^2$ behavior must be... fields.

Interplay between UV and IR, it view a common theme is study of anomalies.

Let me mention some applications - first in QCD

- All we know is the real world QCD confines quarks and spontaneously broken chiral symmetry leading to light N=1 bound

- If $\nu_e = 3$, can prove QCD breaks chiral symmetry anomalously - it's just anomalous matching.

- Decay rate $\pi^0 \rightarrow \gamma \gamma$ determined by $\Lambda_{\text{QCD}}$.
Certain H - H couplings are determined by
unremovable -like vacuum terms.

More generally, in 4d

In the symmetric model of partons UV scales
some group theoretic numbers are fixed by
ς - 2
terms dictated by anomalies,

anomaly in 1/2, anomaly in 1
but not

Extended in 2 - 1 \times 2 \times 1 - leptogenesis.

More generally, proper parts are useful in the
study of dynamical\* strong coupling dynamics and
provide constraints on quark exchange,

i.e., in Seiberg duality.

\* The Standard Model gauge chiral symmetries
that distinguish L from R handed fields

i.e., (Y), x 2, \bar{a} \text{ scalar, each of such.}

In general such theories are inconsistent because
\text{gauge Intr.} is violated at the quantum level, D = 4

\[ S^n \times X \quad a \text{ Tr} [W_3] = \tau^{abc} \quad = 0 \quad \text{for real rep.}

\text{left-handed fermion to } \mathfrak{su}(n), \mathfrak{so}(n), \mathfrak{so}(n+2), \mathfrak{f}(n/2) \quad n \geq 3

\text{in rep } \Gamma

\text{in complex reps}
SM: \[ G = SU(3) \times SU(2) \times U(1) \]

and GUTs: \[ G = SU(5), SO(10), E_6 \]

we have potential anomalies

\[ \Rightarrow \] anomalies restrict model building

Anomalies in other dimensions.

In 20 dimensions have chiral fermions and there is a chiral anomaly

\[ \text{For fermions in rep } \mathbf{F} \text{ of group } G \quad \text{ associated to gravity, the chiral anomaly is} \]

\[ \text{Fermion } \mathbf{F} \]

\[ \mathrm{ch}(F) = \text{Tr} \quad \text{Chern character} \]

\[ \mathbf{F} = \text{A root given by the term} \]

\[ \Rightarrow \]

these are objects called characteristic classes, which when integrated give topological

invariants of vector bundles.
One can also have chiral bosons which contribute to the anomaly.

\[ d=2 \quad \text{chiral bosons} \quad \phi = \pm \phi_{\mu} \]

\[ d=6 \quad \text{chiral tensor fields} \quad B_{2}, \quad H_{3} = \partial \phi B_{2} \]

\[ H_{3} = \pm \times H_{3}. \]

Frequently, in \( d>10 \), we no longer consider the coupled gauge and gravitational anomalies only for \( \Sigma = \mathbb{R}^d \times \mathbb{R}^d, \) spinor fields \( \mathbb{C} P_{d+1} \) are needed.

Anomalies in Defects and Boundaries.

In QFT physics there is an effective description using CS theory:

\[ \int \omega_{d+1} = \int \text{Anomaly} \quad \text{for} \quad U(1) \text{ gauge theory} \]

Gauge variation \( \delta \int \text{Anomaly} = \int \text{Anomaly} \)

\[ H_{3}, \quad H_{3} \]

\[ \delta \int H_{3} \left( \wedge \nabla \right) = \int \text{Anomaly} \quad \text{Stokes} \quad \text{stokes} \]

\[ H_{3}, \quad \text{and} \]
To have a gauge invariant description, there must be another contribution to the anomaly localized on $\Sigma_{3}$.

\[ \text{Charged edge states of IQHE} \]

In general, there is a mathematical relation

\[ \lambda_{m} = d \lambda_{m-1} \quad \text{and} \quad \lambda_{m} = d \lambda_{m-2} \]

\[ \text{charged anomaly} \quad \text{form} \quad \text{gauge gauge} \quad \text{anomaly} \quad \text{in} \quad d-2 \quad \text{dimension} \]

i.e., $F \wedge F = d\omega_{2}$, $d\omega_{2} = d(\lambda dA)$ for $d = 4$.

and this can be manifested physically for certain kinds of defects in localized channel $z$ and hence anomaly in low-energy effective description.
"Anomaly inflow" w/ C. Gell-Mann

Bulk theory w/ CS couplings involving fields coming charge of detect

Detect w/ Churel

\[ \text{v} \rightarrow \text{unbound} \]

E.g. M theory w/ "axion strings" X-ray

\[ \text{f}_\text{fract} \rightarrow \text{v} \text{ev} \]

\[ \theta^2 \rightarrow \delta^2 \text{ (v axion)} \]

Has many implications in D-brane physics

where a class of couplings can be determined by anomaly inflow from intersecting D-brane

with Green model

\[ \text{Chiral fermions on intersection} \quad \text{MS-brane} \]

Also play an important role in M-theory
Condensed matter applications

I want to learn but a quick search shows physics involving anomalies now.

QHE

Topological insulators

Weyl metals

Chiral magnetic effect

Hydrodynamics of anomalous currents.

In general, the recovery tends to be non-quantitative fields of many systems and finite dimensions are often dictated by chiral, gauge and gravitational anomalies.