Are Galileon Theories Supersymmetrizable?

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Outline

- Goal: Classify and understand properties of low-energy EFTs arising from spontaneous symmetry breaking
- Method: Use on-shell tree-level amplitudes and their soft behaviour
A 3-brane in a 6-d space

Transverse directions $x_4, x_5$, longitudinal directions $x^\mu = (x_0, x_1, x_2, x_3)$

Embedding of the brane is described by two real scalar fields

$\Rightarrow$ one complex scalar field $\phi$
Spontaneously Broken Symmetries of $\phi$

6-d Poincaré group is spontaneously broken $\Rightarrow$ two Goldstone bosons $\phi$, $\bar{\phi}$

- Translations $P_4$ and $P_5$, e.g. $\phi \rightarrow \phi + a \partial_4 \phi$
  $\Rightarrow$ Non-linearly realized as a shift symmetry $\phi \rightarrow \phi + c$

- Lorentz rotations $M_{\mu 4}$ and $M_{\mu 5}$
  $\Rightarrow$ Non-linearly realized as a spacetime-dependent shift symmetry $\phi \rightarrow \phi + c + v^\mu x_\mu$

Not obvious from the Lagrangian.
At leading order, only the brane tension term contributes

$$\mathcal{L}_{DBI} = \sqrt{-\det h_{\mu \nu}} = \sqrt{\det (\eta_{\mu \nu} - \partial_\mu \phi \partial_\nu \bar{\phi})}$$

where $h_{\mu \nu}$ is the induced metric on the brane.
DBI and Galileon

Schematically,

\[ \mathcal{L}_{\text{DBI}} = \int d^4x \left( \frac{1}{2} \partial^2 \phi \bar{\phi} + g_4 \partial^4 \phi^2 \bar{\phi}^2 + g_6 \partial^6 \phi^3 \bar{\phi}^3 + \cdots \right) \]

- Shift symmetry is ‘trivial’
- Space-dependent shift symmetry is ‘non-trivial’

Galileon theories

- Sub-leading terms to brane action
- Second order EOM
- Decoupling limit

\[
\begin{align*}
\mathcal{L}_3 &= \partial^4 \phi^2 \bar{\phi} + \text{h.c.} \\
\mathcal{L}_4 &= \partial^6 \phi^2 \bar{\phi}^2 \\
\mathcal{L}_5 &= \partial^8 \phi^3 \bar{\phi}^2 + \text{h.c.}
\end{align*}
\]
Soft Behaviour

Symmetry

\[ \phi \rightarrow \phi + c \]

\[ \phi \rightarrow \phi + c + v^\mu x_\mu \]

Soft behaviour

\[ A_n(p_\phi) \sim p_\phi \text{ as } p_\phi \rightarrow 0. \]

\[ A_n(p_\phi) \sim p_\phi^2 \text{ as } p_\phi \rightarrow 0. \]

SSB of supersymmetry \( \Rightarrow \) Goldstino \( \psi \)

Non-linear realization of broken SUSY is a shift \( \psi \rightarrow \psi + \eta \), i.e.

\[ A_n(p_\psi) \sim p_\psi \text{ as } p_\psi \rightarrow 0 \]

Aim: Soft behaviour of amplitudes \( \Rightarrow \) Existence of SUSY Galileons
Supersymmetry

$\mathcal{N} = 1$ DBI has

- Soft behaviour $p^2_\phi$ for $\phi$, $\bar{\phi}$
- Soft behaviour $p_\psi$ for partner Goldstino
  $\Rightarrow$ Must break $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$

What about the Galileon?

- Start with $\mathcal{L}_{\text{Gal}} = g_3 \mathcal{L}_3 + g_4 \mathcal{L}_4 + g_5 \mathcal{L}_5$
- Field redefinition $\phi \rightarrow \phi + a(\partial \phi)^2$
- Choose $a$ so that $\tilde{g}_3 = 0$, i.e. $\mathcal{L}_{\text{Gal}} = \tilde{g}_4 \mathcal{L}_4 + \tilde{g}_5 \mathcal{L}_5$
- $\mathcal{N} = 1$ SUSY $\mathcal{L}_4$ in the next talk!

We answer the question of supersymmetrization of $\mathcal{L}_3$ via $\mathcal{L}_4$ and $\mathcal{L}_5$. 
Supersymmetrizing the Quintic Galileon $\mathcal{L}_5$

- Find all structures for $\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5)$
  - Lorentz invariance $\Rightarrow$ function of spinor brackets
  - Locality $\Rightarrow$ polynomial
  - Little group $\Rightarrow$ does not scale
  - Fermi-Bose symmetry $\Rightarrow$ symmetric under $(1 \leftrightarrow 3), (2 \leftrightarrow 4), (3 \leftrightarrow 5)$
  - Mass dimension $\Rightarrow [\mathcal{A}_5] = -9$

- Impose soft behaviour $p^2_\phi$

$$\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5) = \tilde{g}_5 [s_{13}s_{25} (s_{15}s_{23} + s_{12}s_{35} - s_{13}s_{25}) + (1 \leftrightarrow 5) + (3 \leftrightarrow 5)] + (2 \leftrightarrow 4)$$
Impose the SUSY Ward identities

$$A_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\psi}_4, \psi_5) = \frac{[25]}{[24]} A_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5)$$

Has a pole as $[24] \to 0! \Rightarrow$ not local

There exists no $\mathcal{N} = 1$ SUSY $\mathcal{L}_5$ with $p^2_\phi$ soft behaviour

$\Rightarrow$ there is no supersymmetrization of $g_3\mathcal{L}_3$
A SUSY quintic Galileon

- What about a theory having $p_\phi^2$ behaviour for the real part of the complex scalar $\phi$, but $p_\phi$ behaviour for the imaginary part?
- We find a consistent solution.

- Does such a theory make sense?
- $p_\phi^2$ scalar is a Goldstone boson from 5d Poincaré group breaking
- $p_\phi$ scalar is an R-axion
Galileons arise as terms subleading to DBI in brane actions
We address the question of possible supersymmetrizations of Galileon theories via on-shell amplitudes
We make definite statements of non-supersymmetrizability of $\mathcal{L}_3$ and $\mathcal{L}_5$
We find evidence that supports the existence of a SUSY $\mathcal{L}_5$ with an R-axion

Thank you!